THE COUPLING OF SHEAR WALLS

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ABSTRACT

In this first project of a research program, initiated to investigate the behaviour of coupled shear walls subjected to seismic type lateral loading, the strength and behaviour of the coupling beams is examined.

The limitations of the elastic "laminar analysis" are examined and the technique is extended so as to predict the elastoplastic behaviour of a cracked shear wall structure and, in particular, the ductility requirements for coupling beams.

Twelve approximately ¾ full size relatively deep reinforced concrete coupling beams, with various aspect ratios and web steel contents, have been tested under static one-way and near-ultimate cyclic loading. The experiments, which attempted to trace every aspect of the behaviour, revealed numerous features not encountered with normal shallow reinforced concrete beams. Shear and flexural failure mechanisms were identified and the nature of deterioration with cyclic loading was observed.

A theoretical approach is suggested which satisfactorily approximates the loss of stiffness of coupling beams after diagonal cracking. Experiments and theory both indicated a loss in excess of 80%.

The significant findings of the experimental investigation have been translated into design recommendations for coupled shear wall structures.

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NOTATIONS

Linear Dimensions

b	width of a rectangular beam
d	effective depth of a beam
d'	distance from the centroid of flexural reinforcement to adjacent edge of beam
h	floor height
l	distance between centroidal axes of coupled shear walls *or* clear span of test beam proper
l_1, l_2	distance from axis of coupled shear wall structure to centroidal axis of Wall 1 and Wall 2 respectively *or* lengths of the diagonal members of an analogous truss
l_s	length of a stirrup
l'	length of a strut in an analogous linear arch
s	distance between inner faces of coupled walls *or* stirrup spacing
x	distance from the top of a wall *or* along a test beam
x_1, x_2	location of a stirrup from one or the other end of a beam
x'	ordinate along a wall referring to a point situated half way between coupling beams
z	internal lever arm
D	overall depth of a beam

Areas and their Properties

a_v	area of one stirrup
A	area
A_s	area of flexural steel in one face of a beam
A_h	area of horizontal web reinforcement
I	moment of inertia
I_1, I_2	moment of inertia of Wall 1 or Wall 2
I_o	$= I_1 + I_2$

I_x	reduced moment of inertia of coupling beam allowing also for shear deformation
I'_x	equivalent reduced moment of inertia of a cracked beam

Forces

$p(x)$, $p(\xi)$	laminar separation force per unit length
p_x	equivalent stirrup force per unit length at x
p_c	the maximum value of p_x
p_o	the minimum value of p_x
p_s	the mean value of p_x
$q(x)$, $q(\xi)$	laminar shear force per unit length
q_u	ultimate laminar shear force per unit length
A	shearing force carried by interlocking aggregates
C	compression force in a beam
D_1, D_2	dowel forces across the flexural reinforcement
S	stirrup force
T	tension force
$T(x)$, $T(\xi)$	axial forces generated in coupled shear walls
T_t, T_b	tension in the top and bottom flexural reinforcement respectively
T_m	maximum tension generated in the flexural reinforcement of a test beam
T_c	tension in flexural reinforcement at midspan of a coupling beam
T'	tensile force in the horizontal web reinforcement
V	total external shear force
V_1, V_2	final shear forces on Wall 1 and Wall 2
$V_{1,o}$, $V_{2,o}$	shear force induced by external load only in Wall 1 and Wall 2
V_d, V_{do}	dowel force acting across the flexural steel in the tension and compression zone respectively
V_c	shear force resisted by beam without web reinforcement

V_s	shear force resisted in a beam by stirrups only
V_a	component of shear force resisted by arch action
V_p	shearing force generated in walls by separation forces
V_u	ultimate shear force across a coupling beam
W	external lateral load, usually of triangular shape, on coupled shear wall structure
W_1, W_2	proportion of external load acting on Wall 1 and Wall 2 respectively
W_e	external load at elastic limit of structural behaviour
W_u	total external triangular ultimate load on coupled shear wall structure
W_y, W_y'	external triangular load which causes all laminae to yield
$\Delta W'$	triangular load increment to cause Wall 1 to attain its ultimate capacity
$\Delta W''$	triangular load increment to cause Wall 2 to attain its ultimate capacity
P, P_i	load on a test beam at increment i
P_u	ultimate load carried by a beam <u>or</u> a point load corresponding with W_u
P_u^*	theoretical ultimate load on a beam
P_o	singular separation force acting at the top of coupled shear walls
$\Delta P'$	a point load increment corresponding with $\Delta W'$
$\Delta P''$	a point load increment corresponding with $\Delta W''$
P_b	axial force in coupling beam resulting from accumulated separation forces

<u>Bending Moments</u>

M_i	moment on a test beam at increment i
M_o	total external cantilever moment
$M_{1,o}$, $M_{2,o}$	moments induced by external load only in Wall 1 and Wall 2
M_p	moments generated in walls by separation forces

$M_{q,1}$, $M_{q,2}$	moments generated by laminar forces in Wall 1 and Wall 2
M_1, M_2	final bending moments on Wall 1 and Wall 2
$M_{1,u}$, $M_{2,u}$	ultimate moment capacity at the base of Wall 1 and Wall 2

Stresses

f_c	concrete compression stress
f'_c	cylinder crushing strength of concrete
f'_{cu}	cube strength of concrete
f_s	steel stress
f_y	yield strength of steel
f_u	ultimate strength of steel
v	nominal shear stress = V/bd
v_{dc}	nominal shear stress at onset of diagonal cracking
E	Young's modulus
E_c	modulus of elasticity for concrete
E_s	modulus of elasticity for steel
G	modulus of rigidity

Displacements

d_a	differential displacement of coupled shear walls owing to axial forces
d_b	laminar deflection caused by flexure and shear
d_f	laminar deflection owing to laminar flexure
d_m	laminar displacement caused by the flexural rotation of coupled walls
d_s	laminar displacement cause by shear
y_1, y_2	lateral deflection of Wall 1 and Wall 2
y_o	lateral deflection at the top of the shear wall structure
y'_o	lateral deflection at top of structure caused by $\Delta W'$
y''_o	lateral deflection at the top of structure caused by $\Delta W''$

y_e	lateral deflection of shear walls at the elastic limit
y_{oe}	lateral deflection of the top of the structure when the elastic limit is attained
y_{py}	lateral deflection of walls at full plastification of laminae
y_{opy}	lateral deflection at the top of the structure at full plastification of laminae
Δ_a	shear displacement owing to arch action
$\Delta_1, \Delta_s, \Delta_2$	components of the shear displacement Δ_v
Δ_v	shear displacement associated with an analogous truss
Δ_s', Δ_s''	elongation of the top and bottom flexural reinforcement respectively over half the span of a coupling beam
Δ_H	elongation of test beam
Δ_H'	theoretical elongation of test beam proper

<u>Rotations</u>

θ_a	beam rotation owing to arch action
θ_B	total laminar rotation
θ_h	rotation in plastic wall hinges after the attainment of the ultimate load
θ_1	beam rotation owing to the elongation of the flexural reinforcement
θ_m	flexural rotation of coupling beams
θ_p	plastic laminar rotation
θ_p'	plastic laminar rotation increment caused by $\Delta W'$
θ_{ph}	plastic laminar rotation caused by θ_h
θ_{pr}	plastic laminar rotations generated by rigid body rotations of coupled walls
θ_w	elastic wall rotations
θ_w'	elastic wall rotation increments caused by $\Delta W'$
θ_w''	elastic wall rotation increment caused by $\Delta W''$

θ_{wr}	rigid body rotation of shear walls
θ_y	maximum elastic laminar rotation at the onset of yielding
Φ_L, Φ_R	rotations of the left and right hand support sections respectively in a test beam

<u>Parameters, Factors and Coefficients</u>

a	a factor containing the ratio A_s/A_h
f	a form factor associated with shear distortions
h	modular ratio, E_s/E_c
p	flexural steel content
p_w	web steel content
C	a geometric factor for coupled shear walls
C_H	a dimensionless factor related to beam elongation
Z	a stiffness ratio
α	a geometric parameter (dimension = 1/Length)
β	a geometric parameter (dimensionless)
γ	a geometric parameter (dimension = 1/Length^3)
ρ	a load ratio. P/W
η	the ratio of the steel force carried by the web reinforcement to the load on the coupling beam
ν	Poisson's ratio <u>or</u> ratio of the beam's span to the length of the stirrups
ν_1	ratio of the beam's length to the internal lever arm
ξ	specific distance x/H or x/l
ξ_e	location of the first yielding lamina
φ	capacity reduction factor

CHAPTER ONE

INTRODUCTION

1.1 Shear Wall Structures

Structural engineers have long recognised the usefulness of certain walls in the overall planning of a multistorey building. When external or internal walls are placed in advantageous positions they can be extremely efficient in resisting lateral loads originating from wind or earthquakes. Provided that they are suitably connected to the floors of the building, that they possess sufficient strength, and as cantilevers they are effectively restrained at their foundations, they can accept a major part of, if not all, the lateral load on a building. By doing so they can considerably relieve those structural units which primarily carry gravity loads. Because they often receive a significant proportion of the accumulated lateral forces (i.e. shear), particularly at the lower floors of a multistorey building, they are collectively termed as "shear walls" in the English speaking world.

Functional and often structural requirements make the use of shear walls mandatory in many buildings. Windowless boundary walls, exterior walls with small openings, permanent interior partition or fire walls, and cores encasing lift shafts and stair wells are only a few of the uses of shear walls. More often than not, such walls are pierced by numerous openings for windows, doors, service holes and other purposes. The structural engineer is fortunate enough if these openings are arranged in a systematic pattern.

It is difficult to clearly define the division between a shear wall with openings (pierced shear walls) and a rectilinear frame with deep members. A few years ago regular shear wall structures, in which two or more walls are separated by vertical rows of openings, began to be referred to as "coupled shear walls". The "coupling" consists of a number of blocks, panels or beams formed between vertically arranged openings. Their contribution determines the nature and degree of interaction amongst the coupled walls.

Though coupled shear walls have been accepted for quite some time as essential structural units of multistorey buildings, our understanding of their behaviour is not even approaching that of beam-column frames. Only in the last decade does an increased interest in them, and a corresponding research activity, emerge from the technical literature. The initiative of the University of Southampton, in organising the first international symposium on tall buildings and shear wall structures in 1966, was a highlight in the sporadic efforts to promote knowledge in this field.*

1.2 The Problems Associated With Coupled Shear Walls

The problems of shear wall structures arise in several ways. Multistorey buildings have become taller and more slender. With this trend the analysis of shear walls often emerges as a critical design item. New construction techniques, such as lift slab construction, often rely entirely on the lateral stability and strength of shear walls. It became evident that the well developed methods of structural analysis, used in the design of rigid jointed frames, are inadequate.

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* See references numbered 45, 46, 48 and 55.

Deformations owing to axial and shear stresses, which were thought of as being insignificant in comparison with flexural deformations, had to be accounted for. This necessitated the search for new techniques of analysis with a manageable demand on computational efforts in a design office.

A particular difficulty arose from the interaction of shear walls and rigid jointed frames. It is not easy to satisfy the requirements of compatibility for two structures, so very different in their behaviour, without the use of an electronic computer, unless one is satisfied with rather crude approximations.

Deep structural members, such as coupling beams, no longer obey the laws of classical flexural theory. To avoid cumbersome stress functions in the assessment of the nonlinear stress pattern in deep members, more attention was directed towards photoelastic and structural model studies. Unfortunately the validity of the results of these are often restricted to a specific structure.

Theoretical or experimental results, however successful the projects are from which they originated, still need to be related to the structural material to be used, which, with a few exceptions, is reinforced concrete. This in turn opens another source of problems associated with the relationship of the classical elastic analysis and the behaviour of cracked reinforced concrete members.

There is little known about the evaluation of the ultimate strength of deep reinforced concrete members subjected to alternating loading. The forces on shear walls may also give rise to unusual combinations of bending moment, axial force and shear force. The laws of such interactions, particularly at the ultimate state, are not known as yet.

The stiffness characteristics, necessary for the determination of the deformations and dynamic properties of a cracked shear wall structure, can only be crudely estimated. The nature of the postelastic behaviour, in particular the demand and availability of ductility in coupled shear walls, is, to the writer's knowledge, wholly unexplored. A lack of understanding in this field may have serious consequences. The damage that can result from a violent earthquake disturbance in a shear wall structure is illustrated by the view of a 14 storey building which survived the 1964 Alaska earthquake in Anchorage. See Fig. 1.1, Fig. 1.2, Fig. 1.3 and Fig. 1.4.

1.3 The Aims of a Research Program

It is hoped that in a long term research program, initiated by the writer in the Department of Civil Engineering of the University of Canterbury, it will be possible to advance our understanding of the behaviour of shear wall structures. It is intended to restrict the work to multistorey reinforced concrete structures and to their performance in situations likely to be encountered in severe seismic disturbances. It is also hoped that with a speedy evaluation of the program it will be possible to make the results available, in the form of design recommendations, to the engineering profession for use and comment. This thesis, dealing with the coupling of shear walls, is the outcome of the first project of the proposed research program.

1.4 The Scope of the Project

In a review of the relevant literature the origin and the development of the analyses of coupled shear walls are traced. In particular the postulates of the so called "laminar analysis" are examined. As far as possible the essence of the more significant contributions are presented in a chronological order.

In the second chapter the elastic laminar analysis is developed in considerable detail. Certain aspects, overlooked or not emphasised by researchers, are brought to the fore. In order to be able to place this new analytical technique in its proper place, the assumptions upon which it is based are critically examined. The analysis is then extended so as to estimate the elastic behaviour of cracked coupled shear walls. An illustrative example demonstrates qualitatively the influence of cracking and the consequent loss of stiffness upon the static quantities.

As an extension of the laminar analysis a step by step procedure is suggested by which also the postelastic behaviour of two coupled shear walls can be approximated. It is shown how the ultimate strength of the structure is attained through a specified sequence of plastification. The equations, giving the intensity of the significant structural actions, the elastic and plastic deformations, are presented for each stage of the elasto-plastic behaviour. A brief numerical example demonstrates the application of the proposed analysis and highlights the extremely large demand for ductility in the coupling system, when only a relatively modest overall ductility is required.

These two chapters also serve to establish, in terms of the classical theory of elasto-plastic structures, the role of coupling beams in ensuring efficient interaction between the coupled walls. The next task was thus to examine also experimentally how reinforced concrete coupling beams would behave and how they could meet the demand for strength and ductility. The fulfilment of this task forms the major part of this project.

Three groups of four coupling beams have been tested. The span to depth ratio of the beams, the amount of web reinforcement, and the type of loading were the major variables in these experiments. Only for the purpose of distinguishing easily the three series of specimens from each other, they are referred to as shallow, medium and deep beams.

Chapter 6 presents, in some considerable detail, the results of the tests on four medium beams. The experiments attempted to disclose every aspect of the behaviour of coupling beams. Correspondingly the behaviour of the flexural reinforcement over the entire span, and that of the web reinforcement over a major portion of the depth, has been assessed. From these the magnitudes and the positions of the internal resisting forces were evaluated. A comparison is made with the current recommendations of the code of the American Concrete Institute. The compression strains in the uncracked and cracked concrete were also followed in some specimens. As components of the deformations, the rotations, the elongations, transverse expansions and the deflections of the beams were determined during one way and alternating cyclic loading. The crack formation is discussed and the nature of the failure mechanisms is presented. The change of stiffness with cyclic loading, the postelastic deformations and the features of deterioration are also assessed. Wherever possible an explanation is offered for the unusual features of behaviour and at the end of the chapter proposals are made, in the form of an analytical study, for the prediction of the distribution of tension forces along the coupling beams.

In the following two chapters similar findings are presented for the groups of deep and shallow coupling beams. For the latter only a summary of the more important features is given. In the deep beams, with a span to depth ratio of approximately one, horizontal web reinforcement was also provided. In one case confining reinforcement was used to strengthen the critical compression zones of a deep beam.

Chapter 9 presents the analytical assessment of the deformations in coupling beams. It is shown that with a suitable analogy, based on the most probable mechanism of the cracked test beams, the four major sources of the rotations can be determined. From this analogy the stiffness of the cracked coupling beams is evaluated and the results of the analysis are compared with the experimental evidence.

A summary of the theoretical and experimental investigation takes the form of conclusions and recommendations in the last chapter. Suggestions are made for future research on coupled shear walls. The most important conclusions are then translated into design recommendations for coupled shear walls in general and for coupling beams in particular.

1.5 The Presentation of the Results

With few exceptions the experimental evidence is presented in graph form rather than in tables. Wherever possible a comparison is made with existing theories or new propositions. In order to conserve space, only the essential and typical experimental results were reproduced here. However, it is believed that the evidence is sufficient to be entirely convincing within the scope of this project.

To enable the reader to examine easily the results and their quantitative interpretations while reading the text, the numerous figures and photographs have been collected in a separate volume. Also at the beginning of the second volume all tables are assembled.

CHAPTER TWO

A REVIEW OF PREVIOUS WORK ON COUPLED SHEAR WALLS

The unusual dimensions, often encountered in shear wall structures, seriously limit the efficient application of the conventional techniques of structural analysis. It is for this reason that numerous attempts were made to develop analytical methods which fit better the behaviour of this type of structure. The review is restricted to the brief examination of such theories put forward to assess the strength and behaviour of coupled shear walls. Works related to other interesting topics on shear walls, in particular the interaction of a set of shear walls or the interaction of shear walls and rigid jointed frames, are beyond the scope of this review.

An approach, by which the statically indeterminate problem of coupled shear walls can be reduced to a relatively simple analysis, seems to originate from Chitty[1]. She studied the behaviour of a number of parallel cantilevers which are rigidly interconnected by cross-bars. These bars are replaced by an equivalent continuous elastic medium which is capable of transmitting the same actions as the cross-bars. The device enables the various actions to be expressed by continuous functions along the cantilever beams. Chitty correctly assessed the equilibrium and compatibility requirements but she neglected the effects of shear. She proposed a differential equation in terms of a continuously varying moment applied by the connecting medium to the cantilever beams. The solution of the problem is completed by satisfying the boundary conditions for the cantilever structure.

The approximation in Chitty's approach consists of the replacement of discrete connecting bars by a set of infinitesimal ones spaced at infinitesimal distances from each other along the cantilevers. Each such bar or "lamina" acts independently. The continuous elastic connecting medium may therefore be thought of as a continuous laminar system between two adjacent cantilevers.

The larger the number of discrete bars to be replaced by the laminar system the more satisfactory results the approximation is likely to furnish. In coupled shear walls of multi-storey building generally one connecting beam is made at each floor. (See Fig. 3.1 and Fig. 3.2). Therefore it may be said that the larger the number of storeys the more useful the laminar analysis becomes.

Chitty and Wen-Yuh-Wan directly applied their studies to multi-storey structures.[2] They compared the results obtained from slope-deflection equations and the laminar analysis and found a satisfactory agreement. The axial deformations in the columns of the single bay multi-storey frames were neglected. For this reason the proposed technique has serious limitations when applied to shear wall structures, in which axial deformation may significantly affect the behaviour.

A very clear review of the approach used by Chitty and Wen-Yuh-Wan, as applied to open web structures, was given by Pippard[3].

In 1952 Green[4] introduced an approximate analysis applicable to frames with deep beams. His approach was similar to the "portal method" which was extensively used to determine the approximate actions in building frames subjected to lateral static loading. He too neglected axial deformations in columns but he emphasised the significance of shear deformations in deep members.

A most penetrating work on the elastic behaviour of coupled shear walls originates from Beck. His first publication[5] deals with wall panels containing one or more rows of openings. The Vierendeel girder, with deep horizontal and relatively slender vertical members, is one of the examples on which he introduced his method of analysis. The individual columns were replaced by an equivalent laminar system, similar in nature to that used by Chitty some ten years earlier. Beck was unaware of Chitty's work.

In his work on simply supported Vierendeel girders, Beck ignored the effect of axial stresses in the horizontal members and the shear distortions in the elastic connecting laminae. Only for the particular structures of his choice was this a justifiable assumption. His differential equation was set up in terms of the laminar shear force. In a number of examples he compared the results of:

(a) A conventional "exact" elastic analysis, which considered the uniform properties of all members as being concentrated at the centre lines between node points.

(b) An "exact" elastic analysis based on the assumption that the columns of the Vierendeel girders deform only over their clear span.

(c) The proposed "laminar" analysis.

This study showed an excellent agreement between the results of (b) and (c) type analyses, even for cases when only five columns connected the top and bottom chords of the Vierendeel girders. It was also revealed that the conventional frame analysis, (a), when not taking into account "deep member effects" could furnish excessively erroneous results. The computational effort is considerably reduced in the laminar analysis.

In an extract of a dissertation[6] Beck presented in 1958 his approach in a more general form. The aim of his study was to replace a very large number of statically indeterminate quantities by a few "mathematically sensible" functions. Instead of setting up a large number of simultaneous linear equations he dealt with a few simultaneous differential equations. In many cases, and the problem of two coupled shear walls is one of these, the solution is obtained from a single second order differential equation. The larger is the degree of static indeterminancy the larger is the appeal of the laminar analysis.

In the same year Zbirohowski-Koscia introduced an approximate lateral load analysis[7] in which the assumptions of the laminar system were used. The advantage of reducing the amount of computational work was lost however when discrete laminar forces were used. The effect of foundation rotations upon the behaviour of coupled shear walls was also considered by making use of the Winklerian concept of elastic foundation response.

In 1959 Beck extended his laminar analysis of Vierendeel girders so as to allow for the significant axial deformations in coupled shear walls and the shear deformations in the coupling beams[8]. In a series of diagrams he clearly demonstrated the limitations of the conventional frame analysis when applied to shear wall structures and also the limitations of the techniques usually applied to castellated beams. At that time he examined symmetrical structures only.[9]

Chapuis and Latil attacked the problem in a more conventional manner[10] by setting up simultaneous equations, equal in number to the number of storeys. This was only possible because the axial deformations were neglected and because only symmetrical shear wall structures were considered. They also proposed an approximate analysis for shear walls containing more than one row of openings.

A similar approach was presented by Albiges and Goulet in 1960 for the analysis of a set of a set of solid shear walls.[11] In the second part of their work they presented an approximate analysis for two coupled shear walls. According to the magnitude of parameter, which expressed the efficiency of shear transfer from one wall to the other, they classified these structures into three groups, as follows:

a.) Walls with small openings (i.e. large coupling beams) in which the deformations owing to shear transfer can be neglected. The analysis is thus based on the Bernoulli-Navier strain hypothesis and linear elastic behaviour.

b.) The wall openings are so large that the contribution of the coupling beam can be neglected. The load is thus resisted by cantilever walls only.

c.) A special analysis is warranted for walls with intermediate openings in which the coupling beams are capable of transferring considerable shear, while being subjected to appreciable deformations.

It is this latter group of shear walls for which Albiges and Goulet developed essentially the same technique as that of Beck[8,9]. They appear to have been unaware of Beck's and Chitty's work. They paid due attention to flexural and axial deformations but they ignored the shear distortions of the coupling beams. The function for the statically indeterminate quantity is given in terms of the laminar shear. In Cahpter 3 it will be shown that this solution is insufficient for nonsymmetrical shear wall structures. "Separation forces" in the coupling beams are also generated and these affect the distribution of the external shear between the two coupled walls.

It may be said that further work on the laminar concept of shear wall analysis was largely influences by the publications of Chitty, Beck, Albiges and Goulet in the English, German and French speaking countries respectively.

In 1960 appeared the first publication of Rosman[12], who is the most prolific theoretical worker on the topic of coupled shear walls. Making use of the laminar system and by strain energy considerations, he established the fundamental Eulerian differential equation of the problem. He chose the solution in terms of the axial force on the walls and expressed this by trignometric series. By applying this approach to a shear wall with two vertical rows of openings, he showed that simultaneous, nonhomogeneous, second order differential equations with constant coefficients, yield the required static quantities. In this rather original mathematical approach, Rosman neither allowed for shear deformations nor considered the separation forces which are exerted by the coupling beams.

In a discussion of Rosman's paper, Beck[13] pointed out the similarity to his mathematical model and showed the complete solution by giving expressions for the laminar separation forces and the singular separation force at the topmost lamina.

Neumann and Walter reported on an experimental comparison with Beck's and Rosman's analysis in the following year.[14] A photoelastic model study of a ten storey symmetrical coupled shear wall structure showed excellent agreement with the theoretical approach. It also indicated that at the location of the coupling beams, where discrete moments are introduced into the shear walls, the stresses do not change abruptly in the outer fibres of the walls. The fringe pattern showed the usual stress concentrations at the reentrant corners.

In the process of establishing the principles to be used in the analysis of any number of coupled shear walls, Erikson and Malmstrøm presented in 1961 the most complete summary of Beck's and Rosman's work[15]. They gave the complete solution for two uniform coupled shear walls subjected to continuously varying lateral loading.

Schulz attacked the problem[16] also by using a continuous connecting medium between the walls, and expressed his fundamental differential equation in terms of the accumulated laminar shear. He neglected the shear distortions and overlooked the existence of separation forces.

The governing differential equation was formulated in terms of the slope of the shear walls by Cardan[17] who, in his analysis, ignored the axial deformations in the walls. This normally leads to excessive errors. He considered the load patterns commonly used to replace seismic forces and also investigated the effects of foundation rotations.

Inspired by Beck's work Mann also proposed an analysis of Vierendeel girders and coupled shear walls[18]. Instead of obtaining an explicit solution in terms of a single function, he employed a finite difference analysis. This is better suited to cope with discontinuities, but for shear wall structures it involves lengthy computations. To keep this analysis relatively simple, Mann neglected significant deformations. This limits the applicability of his technique.

The shear wall studies of Tomii bear no relation to any work previously reported. In numerous experimental projects, which extended over a period of more than ten years, Tomii determined the load-deformation characteristics of various shear wall panels at Kyushu University. In some of his investigations, he considered the behaviour of a whole wall containing one or more openings, rather than the behaviour of wall components. As would be expected, he encountered considerable scatter in his test results. Tomii formulated a relatively simple empirical relationship between parameters representing the load and the relative size of the openings[19]. The type of shear wall, rather popular in Japan, is contained in a boundary frame. So the properties of the frame, the wall and the size of the openings affect the behaviour of the structure. Surprisingly he observed that the position of the openings did not greatly affect the performance. He expressed the relative sizes with the following parameter:

$$\xi = \sqrt{h_o l_o / h' l'}$$

where h_o and l_o = the height and length of the opening

h' and l' = the height and length of the boundary frame measured at its inside.

The effect of the load was incorporated in another parameter:

$$\eta_R = \frac{Q - Q_F}{Q_o - Q_F}$$

where Q = total applied shear force

Q_F = shear force applied to the frame only

Q_o = shear force applied to the shear wall only

when each of the three structures was subjected to the same shear displacement. The Q terms in the above expressions may be considered as the appropriate shear stifnesses.

Tomii found that the value of η_R did not change appreciably while the load, Q, was increased to ultimate and that the effect of openings could be predicted by the following expression:

$$\eta_R = 1.5(1 - \xi)^2$$

This fitted best his experimental results.

From another series of small scale models of two-storey shear walls, containing symmetrically placed rectangular and round openings, Tomii found[20] that the shearing deformation of walls with small openings was essentially the same as that of the walls without openings, when ξ was less than .5. With increasing size of the holes the models behaved more and more like reinforced concrete frames.

The crack pattern observed by Tomii on semi full size two storey shear wall structures, with two rows of major openings and some duct holes, illuminated their behaviour[21]. The wall elements were also subject to additional vertical load so that the structure could represent the bottom two storeys of a five storey building.

Stiller[22] carried out a theoretical study of the stresses induced in shear wall structures consisting of closed and open sections. He introduced functions which enable the maximum stresses at the edges of a panel, containing an opening, to be expressed in terms of the stresses generated in homogeneous elastic shear wall elements without openings. In particular, he investigated the effects of torsion in core type shear walls. He also examined experimentally the weakening effect of holes in shear walls. A row of rectangular openings was replaced by circular holes of an equivalent area. This enabled Stiller to reduce the stress concentrations at the corners and to obtain a more uniform stress pattern in his photoelastic models. A comparison of a number of five storey shear wall models with and without circular openings, enabled the stress in the extreme fibres only to be compared. He found that openings affect the shear stresses particularly at the corners of box section shear wall cores.

In 1962 Rosman extended the laminar analysis by considering two coupled shear walls supported on separate foundations.[12] When the settlement and the tilting of the footings is being examined the problem becomes siply a matter of satisfying the appropriate boundary conditions for the fundamental differential equation. In the third part of his theoretical investigation[12], Rosman dealt with coupled shear walls with variable flexural rigidity. To retain the advantages of the laminar analysis it became necessary to assume that the wall properties vary continuously with the height along the structure. Even by assuming a linear variation of wall thickness and Young's Modulus with the elevation of a particular section, the volume of manual computation becomes prohibitive.

Deschapelles showed that finite difference equations may also be used in the laminar analysis.[23] He restricted his study to symmetrical structures.

In the same year Beck presented, in a very clear manner, his theory in English[24] and used symmetrical coupled shear walls for his examples. Unfortunately several authors used this paper as a basis for the study of nonsymmetrical structures also.

Candy and Armstrong were the first ones to point out in the ensuing discussion[25] that cracking, even in a moderately loaded reinforced concrete structure, must account for a loss of symmetry.

Cardan rightly suggested[26] that a general matrix analysis, which allows for all the significant deformations in a coupled shear wall structure can now be easily programmed for a computer.

In a report on observed damage, which resulted from the 1960 Chilean earthquake, Steinbrugge and Flores described[27] the presumed behaviour of a number of shear wall buildings situated in Valdavia. They observed that failure often occurred at vertical or horizontal lines along which openings were situated. Either a row of columns or wall panels failed at the same floor owing to horizontal interstorey shear, or a vertical row of spandrel beams was destroyed owing to high vertical shear induced between adjacent wall-columns. These units often exhibited two major diagonal cracks crossing each other - the familiar features of diagonal tension. The report draws attention to the careful assessment of the behaviour and detailing of these potentially weak links, such as coupling beams, in seismic shear walls.

To verify the degree of approximation involved in the laminar analysis Rosman also undertook a photoelastic examination[28] of a uniformly loaded ten storey symmetrical shear wall with a single row of openings and found that:

a.) there was very good agreement with stresses derived from the laminar analysis;

b.) the discrete effects of the coupling beams caused stress concentrations only over a small area, so that these have no significance in a reinforced concrete structure;

c.) the stresses in the outer fibres of the shear walls vary continuously along the height of the structure and do not appear to be affected by the discontinuities (i.e. openings) within the body of the walls. It is to be noted, however, that Rosman used rather shallow beams in his epoxy resin models.

Two methods of frame analysis, one suitable for manual calculation, the other for processing by a computer, have been compared by Frisenmann, Prabhu and Toppler[29]. The first method consists of replacing all vertical members of a frame by a single equivalent column. This represents the combined stiffness of all columns. The restraints offered by the beams are applied at each floor to the equivalent column. If the number of storeys is large the analysis can be carried out by means of a single function. The other method uses influence coefficients and obtains all quantities by matrix inversion. The authors claimed the usefulness of both these techniques with respect to shear wall studies without having recognised the need to include the distortions which were previously shown to be significant.

In 1964 Arcan reported on a theoretical and experimental investigation of coupled shear walls, carried out in Bucharest[30]. He used the analogy of an inhomogeneous beam containing a central strip, with a low modulus of rigidity, along its length. The actions were expressed in terms of stresses. The strain compatibilities were satisfied at the boundaries of the central strip, which represents the row of openings in a shear wall. The familiar differential equation was established in terms of shear stresses generated in the central strip. Arcan recognised the need for correcting the wall deflections, resulting from external load and the laminar shear forces, in unsymmetrical shear walls, but he assumed that a single force applied at the top of the structure is sufficient to do this. In fact a continuously varying system of laminar separation forces is also required. Arcan expressed the shear stresses induced in the connecting medium, by a similarity to Jourawski's classical equation, in this form

$$q(x) = \psi(x)\frac{Q}{I}$$

where $q(x)$ = shear force per unit height of wall

Q = the first moment of area above the fibre considered taken about the centroidal axis of the section

I = second moment of area of the whole section

$\psi(x)$ = a function which incorporates the external shear pattern and the efficiency of the connecting medium in transferring shear.

On photoelastic models Arcan too verified the close agreement with the laminar analysis. In one test specimen he replaced the connecting beams of the shear walls with a continuous, presumably rubber, medium. From the fringe pattern he obtained essentially the same extreme fibre stresses as in models containing discrete coupling beams.

Arcan pointed out that in reinforced concrete shear walls one should expect a lower modulus of rigidity because of cracking at the corners of the openings. From preliminary tests on reinforced concrete models he obtained a 75% reduction of the modulus of rigidity.

Decaushy[31] continued the theoretical work initiated by Albiges and Goulet[11] in France. He examined the significance of axial deformations in the coupled walls and suggested that in certain cases they may be neglected, whereby the computation is simplified. He devoted a considerable part of his study to the problem of foundation tilting and presented a family of curves as design aids. In 1963 Rosman published three further papers, all of which dealt with the extension of the laminar analysis. In considering the interaction of shear walls and flexible frames[32] he recognised that owing to rotations of the foundations, shear walls do not always participate with full efficiency in the lateral load resistance. To allow for this he postulated a mathematical model which lent itself to the same treatment employed in the laminar system.

For architectural reasons coupled shear walls are sometimes supported on vertical or sloping columns which do not provide full base fixity for the cantilever structure.

After introducing the English readers to his technique, Rosman presented an extension of the laminar analysis for these types of shear wall structures.[33] In his third paper he summarised the highlights of his previous work (in German) and placed more emphasis on structural behaviour.[34]

With one exception all previous publications deal with shear walls with uniform cross sections over their full heights. In 1965 Burns introduced a rather practical approach to the analysis of coupled shear walls with variable cross sections.[35] He assumed that the second moment of area, instead of changing in steps, varies in a parabolic fashion. Following Beck's technique he arrived at a differential equation for which no explicit solution was available. This he overcame by a programmed successive approximation. He summarised his results into design charts which gave the significant parameters corresponding with the load pattern normally employed for seismic design purposes. In a discussion of Burns' work it was pointed out that the basic assumptions with respect to the behaviour of the walls, and in particular to the deformation characteristics of the coupling beams, need be reconsidered in the light of cracking, which must be expected under seismic load conditions.[36]

Without reference to any previous work, Magnus proposed in 1965 a method essentially identical with the laminar concept.[37] His differential equation was in terms of the beam displacements, caused by wall elongations. Important compatibility requirements were neglected.

Balaš and Szabo used Beck's approach when they demonstrated that for n sets of laminar coupling of $(n+1)$ shear walls n simultaneous second order differential equations need be solved.[38] They also made a photoelastic evaluation of the stresses, in short and relatively deep coupling beams.

Coupled shear walls supported on columns is the subject of yet another paper by Rosman[39], in which he processes the laminar analysis by minimising the total complementary energy. He observed that the variation of wall properties with the height had only small effect upon the laminar shear. It is thus likely that the analyses developed could be used with good approximation in most design situations.

The laminar analysis can deal with any type of load pattern as long as this is expressed by a continuous function of the distance from one end of the structure. Rosman also gave the solution for variable wind pressure intensity, i.e. trapezoidal load[40]. He examined two coupled shear walls which were interconnected by an infinitely rigid diaphragm at the top. This situation is frequently encountered in shear cores where the penthouse forms a very rigid connection between the walls. Rosman showed by an example that serious disturbance occurs over the top one third of the structure where stresses, as a general rule, are never critical. The complete solution for this type of framing is also given in Chapter 3.

In 1965 Rosman summed up in a small book most of his previously published work on the laminar analysis of coupled shear walls[41]. This contains a number of design charts. It has become a valuable reference work in Europe where the elastic analysis for lateral load is usually justified.

Small improvements on the existing theories, rather than fundamentally new concepts, were published in 1966 in different parts of the world. Thadani[42] suggested the extension of conventional frame analysis using slope deflection equations applicable to a substitute frame. Dhillon[43] reformulated Chitty's original concept but he has not included the distortions so significant in coupled shear wall structures.

Shear walls interconnected by slabs only were studied by Kazimi[44]. He assessed the stiffness of the participating slab so that it could be replaced by an equivalent coupling beam. He used matrix analysis to determine the static quantities and found good agreement with photoelastic models.

MacLeod examined the stiffness characteristics of a shear wall with a single row of openings and made a comparison with solid shear walls[45]. He used finite element techniques for the theoretical work and aluminium models for his experimental studies.

One storey walls with one or two openings were photoelastically studied by Kokinopoulos[46], who also derived a set of design curves. These were based on the requirement that the tensile strength of the concrete governs the choice of the wall thickness.

A photoelastic experiment of a coupled shear wall resting on two columns was reported by Walch[47]. He obtained good agreement with the theoretical results of Rosman[33].

An experimental project was carried out by Barnard and Schwaighofer[48] to determine the effective width of a slab which acts as a connecting medium between two shear walls. Surpringly they found that the entire width of the slab, which was varied in the experiment, participated in the load transfer. The maximum difference between the experimental results and the results supplied by the laminar analysis was only 5%. They also suggested a method by which the continuously varying laminar shear, as given by hyperbolic functions, can be speedily approximated for design purposes.

Khan highlighted some of the problems encountered by design engineers in the United States[49]. He stated that, from the view of economy, the use of shear walls has become imperative for buildings 30 to 60 storeys high. Khan drew attention to gravity loads which may not be received by the walls proportionally, so that the coupling beams may have to equalise the stresses. An iterative method of analysis with forced convergence was suggested. In fact there is no difficulty in incorporating gravity load effects into the laminar analysis.

Most recently, Coull published a series of studies in which the familiar laminar concept was used. With Choudhury[50] he presented a number of design curves for critical stresses and for the deflection at the top of the structure owing to uniformly distributed load. As a continuation of this they provided the same information for triangular load and a single force[51]. Unfortunately the information is given in a form which is of little use when high intensity (i.e. seismic) loading need be considered. This was pointed out in a subsequent discussion[52] where it was also shown quantitatively that cracking and the consequent loss of stiffness may account for large changes in the static quantities.

A new technique for the analysis of shear walls with stepped sectional properties was introduced by Coull and Puri[53]. The discontinuities which occur in the derivatives of the deflection functions were overcome by the introduction of corrective series. These are claimed to converge rapidly. The results were checked against experiments on araldite models. The agreement for deflections was particularly good when allowance was made for the semirigid connections of the coupling beams. The relaxation of the beam supports, caused by local wall deformations, was compensated for in the analysis by making the effective length of the coupling beam equal to its clear span and its depth.

Gurfinkel extended the cantilever moment distribution method of Grinter and Tsao to Vierendeel frames[54] in such a way that the final moments could be obtained in only two cycles of distributions. When axial shortening need also be considerered, as in the case of coupled shear walls, an additional set of moments need be distributed. The application of the method is restricted to single bay symmetrical multistorey frames.

A number of other publications, also dealing with the analysis, behaviour and strength of shear wall structures, have been reviewed by Coull and Stafford Smith[55].

CHAPTER THREE

THE ELASTIC LAMINAR ANALYSIS

3.1 Introduction

The derivation of the elastic laminar analysis, accounting for all the significant actions and deformations in two coupled shear walls, is presented in some detail. It is largely based on the Beck-Rosman approach. Some details, which are part of the complete solution and which have not been considered or emphasised by previous researchers, are also presented.

In the evaluation of the boundary conditions only two situations, which occur frequently in practice, are considered.

The assumptions, upon which this analysis is based, are summarised and later critically examined. In particular, the likely effect of cracking upon the behaviour of coupled shear walls is discussed. An approximate elastic analysis, which takes the effects of cracking into account, is proposed and its application is illustrated in 3.5.

3.2 The Assumptions

a.) Each element of the structure behaves as a perfectly elastic, homogeneous and isotropic body. The elastic properties of the material are the same throughout the structure.

b.) The Bernoulli-Navier hypothesis holds for the distribution of flexural strains.

c.) The floor heights are uniform so that the coupling beams are equally spaced.

d.) All openings are of the same size.

e.) The sectional properties of the walls do not vary with the height of the structure.

f.) The sectional properties of all coupling beams, but the topmost one, are the same.

g.) The stiffness of the topmost beam is one half of that of the other coupling beams.

h.) The axial deformations in the coupling beams are neglected.

i.) The shear deformations in the walls are neglected.

j.) The stiffnesses of the walls are so much larger than those of the coupling beams, that their slopes are locally not affected by the discrete beams. Consequently the slopes and deflections of the two walls are identical at every point over the height of the structure. Therefore each of the coupling beams will have a point of contraflexure at its midspan.

k.) The external lateral load can be expressed as a continuous function of the distance x, which is measured from the top of the structure. See Fig. 3.1.

3.3 The Model Structure

The structure shown in Fig. 3.1 may be replaced by an equivalent model. In this the discrete coupling beams are substituted by elastic laminae, placed at an infinitesimal distance apart, as shown in Fig. 3.2.a. This substitution is valid if the laminae possess the following properties:

Area of lamina $= \frac{A}{h}\,dx$

Second moment of area of lamina $= \frac{I}{h}\,dx$

where A and I are the area and moment of inertia respectively, of the discrete coupling beams.

For the purpose of analysis, the coupled shear wall structure is cut through the axis which passes through the points of contraflexure of the coupling beams, as shown in Fig. 3.2.b. The internal, statically indeterminate, actions along this cut are as follows:

a.) A continuously varying shearing force, q(x), to be referred to as "laminar shear".

b.) A continuously varying horizontal axial force, p(x). This will be termed a "separation force".

Because of assumption j in 3.2, no moments act along this cut. The laminar shear and separation forces are taken as positive when acting in the directions shown in Fig. 3.2.b.

The two statically determinate cantilever walls must be subject to such a system of laminar forces that the compatibility of deformations along the cut through the coupling system is satisfied.

It may be noted that the separation forces, p(x), have been, with two exceptions[6,15], either ignored or overlooked by all researchers reviewed in Chapter 2.

3.4 The Analysis

3.4.1 Wall Actions and Deformations

3.4.1.1 The external load can be easily distributed between the two cantilevers, so that the compatibility requirements are also satisfied. The pattern of the loading is defined in 3.4.6. As the deflections of the cantilevers are to be the same at every level the load is allotted to each in proportion of its flexural rigidity.

If W = the total external lateral load

V = the total external shear at any level

M_o = the total external cantilever moment at any level. (This must be a continuous function of x.)

I_1, I_2 = the moment of inertia of wall 1 and wall 2

I_o = $I_1 + I_2$

then the external actions (Fig. 3.3.a) are distributed between the two walls in the familiar manner, i.e.

$$W_1 = \frac{I_1}{I_o}W \quad \text{and} \quad W_2 = \frac{I_2}{I_o}W$$

$$V_{1,o} = \frac{I_1}{I_o}V \quad \text{and} \quad V_{2,o} = \frac{I_2}{I_o}V$$

$$M_{1,o} = \frac{I_1}{I_o}M_o \quad \text{and} \quad M_{2,o} = \frac{I_2}{I_o}M_o \qquad (3.1)$$

3.4.1.2 The internal laminar shear, $q(x)$, is responsible for an axial force, $T(x)$, and bending moments, M_q, in the walls. With reference to Fig. 3.3.b, these can be defined as follows:

$$T_1(x) = T_2(x) = \int_o^x q(x)\,dx = T(x) \qquad (3.2)$$

$$M_{q1} = l_1 T(x) \quad \text{and} \quad M_{q2} = l_2 T(x) \qquad (3.3)$$

As a consequence of the axial load, $T(x)$, a section across wall 1 will move upwards by

$$\frac{1}{EA_1}\int_x^H T(x)\,dx$$

The differential displacement between the two walls as illustrated by Fig. 3.3.c. is therefore

$$d_a = \frac{1}{E}\left(\frac{1}{A_1} + \frac{1}{A_2}\right) \int_x^H T(x)\,dx \qquad (3.4)$$

due to axial forces only.

As a general rule the deflections of the two cantilever walls, owing to the moments induced by the laminar shear, are not the same. It follows from the equality of the curvatures

$$\frac{d^2y_1}{dx^2} = \frac{l_1 T(x)}{EI_1} = \frac{l_2 T(x)}{EI_2} = \frac{d^2y_2}{dx^2}$$

that only in the exceptional case when

$$l_1 I_2 = l_2 I_1$$

would deflections in both walls be the same.

To satisfy the compatibility requirements corresponding with the assumption made previously in 3.2.j, it is necessary to introduce correcting moments. These generally overlooked moments can be generated by the application of separation forces, p(x).

3.4.1.3 It is apparent from Fig. 3.3.d that the separation forces form a self-equilibrated system. The actions on each of the two walls are equal and opposite, i.e.

$$V_{p,1} = -V_{p,2} = V_p \quad \text{and} \quad M_{p,1} = -M_{p,2} = M_p$$

3.4.2 The Equilibrium and Compatibility Conditions for the Walls

The equilibrium equations for the cantilever walls may be established so that the compatibility requirements are also satisfied. With reference to Fig. 3.3 the total moments generated in each of the two walls are

$$M_1 = M_{1,0} - M_{q,1} - M_{p,1} = M_{1,0} - l_1 T(x) - M_p \qquad (3.5.a)$$

$$M_2 = M_{2,0} - l_2T(x) + M_p \tag{3.5.b}$$

The induced shear forces are

$$V_1 = V_{1,0} - V_p \quad \text{and} \quad V_2 = V_{2,0} + V_p \tag{3.6}$$

The compatibility requirement is that

$$\frac{M_1}{EI_1} = \frac{M_2}{EI_2} \quad = \frac{d^2y}{dx^2}$$

Hence by substitution from Eqs. (3.5.a) and (3.5.b)

$$(I_1 + I_2) M_p = (I_2 M_{1,0} - I_1 M_{2,0}) + (I_1 l_2 - I_2 l_1) T(x)$$

is obtained. It is recalled that the external moments have already been distributed according to the compatibility requirement, Eq. (3.1). Therefore

$$I_2 M_{1,0} - I_1 M_{2,0} = 0$$

By introducing, for the sake of simplification, the term

$$\frac{I_1 l_2 - I_2 l_1}{I_0} = C \tag{3.7}$$

the moments generated by the separation forces can be determined thus

$$M_p = CT(x) \tag{3.8}$$

They can be eliminated from Eqs. (3.5.a) and (3.5.b), which yield the final moments in the following simple form:

$$M_1 = M_{1,0} - l_1T(x) - CT(x) = \frac{I_1}{I_o}[M_0 - lT(x)] \tag{3.9.a}$$

$$M_2 = \frac{I_2}{I_0}[M_o - lT(x)] \tag{3.9.b}$$

3.4.3 <u>Laminar Actions and Deformations</u>

As a result of the wall deformation discontinuities occur at the ends of the laminae where, for the purpose of this analysis, they have been cut. Consequently the laminar shear must produce deformations which eliminate these discontinuities. The displacements resulting from the different actions at the cut ends of the laminae are as follows:

3.4.3.1 From Fig. 3.4.a. It is evident that the displacement due to flexural deformations of the walls is

$$d_m = l_1 \frac{dy}{dx} + l_2 \frac{dy}{dx} = l \frac{dy}{dx}$$

but $$\frac{dy}{dx} = \int_x^H \frac{M_1}{EI_1} dx = \frac{1}{EI_1} \int_x^H \frac{I_1}{I_o} [M_o - l\,T(x)]\, dx$$

hence $$d_m = \frac{l}{EI_o} \int_x^H M_o dx - \frac{l^2}{EI_o} \int_x^H T(x) dx \qquad (3.10)$$

3.4.3.2 The laminar displacement owing to extensional deformations of the walls is, from Fig. 3.4.b or Fig. 3.3.c, given by Eq. (3.4).

3.4.3.3 By taking twice the free end deflection of the cantilever, the displacement of the laminae owing to flexural deformations only is found from Fig. 3.4.c to be

$$d_f = \frac{hs^3 q(x)}{12EI} \qquad (3.11)$$

3.4.3.4 As the coupling beams are often relatively short, it is necessary to consider also shear deformations. From Fig. 3.4.d and from elementary principles this is found to be

$$d_s = \frac{fhs}{GA} q(x) \qquad (3.12)$$

where f = is a form factor which allows for the pattern of shear stress distribution. For rectangular sections this is 1.2.

$G = E/2(1+\nu)$ is the modulus of rigidity. For design purposes this may be taken as 40% of the modulus of elasticity.

ν = Poisson Ratio

In order to abbreviate further computations it is convenient to combine the latter two deformations into a single term thus,

$$d_b = d_f + d_s = \frac{hs^3}{12EI}\left(1 + \frac{12EIf}{GAs^2}\right) q(x)$$

It is also convenient to introduce an equivalent or reduced moment of inertia for the coupling beams

$$I_x = \frac{I}{\left(1 + \frac{12EIf}{GAs^2}\right)} \tag{3.13}$$

which allows for the shear distortions. Using these simplifications the laminar deflection becomes

$$d_b = \frac{hs^3}{12EI_x} q(x) \tag{3.14}$$

3.4.4 The Differential Equation of Laminar Actions

From the four diagrams shown in Fig. 3.4 it is evident that laminar compatibility is satisfied when

$$d_m - d_a - d_f - d_s = d_m - d_a - d_b = 0$$

Hence by substituting the appropriate terms from Eqs. (3.10) and (3.4) add (3.14)

$$\frac{1}{EI_o}\int_x^H M_o dx - \frac{l^2}{EI_o}\int_x^H T(x)\,dx - \frac{1}{E}\left(\frac{1}{A_1} + \frac{1}{A_2}\right)\int_x^H T(x)dx - \frac{hs^3}{12EI_x} q(x) = 0$$

and by combining the common terms and by differentiating these with respect to x, the following expression is obtained

$$\frac{1}{EI_o}M_o + \frac{1}{E}\left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{l^2}{I_o}\right)T(x) - \frac{hs^3}{12EI_x}\frac{dq(x)}{dx} = 0 \qquad \text{-- 3.14 (b)}$$

As the axial force is the accumulation of laminar shear, it follows from Eq. (3.2) that

q(x)

$$\frac{dq(x)}{dx} = \frac{d^2T(x)}{dx^2} \qquad q(x) = \frac{dT(x)}{dx} \text{ -- (3.2) or } T(x) = \int_0^x q(x)\,dx \qquad (3.15)$$

When the properties of the structure are combined into the following two parameters

$$\alpha^2 = \left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{l^2}{I_o}\right)\left(\frac{12I_x}{hs^3}\right) \qquad (3.16)$$

$$\gamma = \frac{12l\,I_x}{I_o hs^3} \qquad (3.17)$$

and use is made of Eq.(3.15) the differential equation of laminar action is finally obtained from 3.14 (b)

$$\frac{d^2T(x)}{dx^2} - \alpha^2T(x) = \gamma M_o \qquad (3.18)$$

The solution of this equation yields the axial force induced in the walls

$$T(x) = A\sinh\alpha x + B\cosh\alpha x + T_p(x) \qquad (3.19)$$

where $$T_p(x) = \frac{\gamma}{\alpha^2}\left(M_o + \frac{d^2M_o}{\alpha^2dx^2} + \frac{d^4M_o}{\alpha^4dx^4} + \ldots\right) \qquad (3.20)$$

is the particular integral.

To complete the solution for a given situation, the integration constants need to be found from the boundary conditions and the moment function, M_o, need be determined from the external load.

3.4.5 The Boundary Conditions

3.4.5.1 When the coupled walls are fully restrained at their base, as shown in Fig. 3.1, and the assumption stated in 3.2.g is also met, the integration constants are found from the following two conditions:

a.) The axial force at the top of the wall must be zero. Hence

$$T(x) = 0 \quad \text{when} \quad x = 0$$

b.) At the base of the walls no rotations occur, hence the lowest lamina does not deform and no actions are induced in it. Hence

$$q(x) = \frac{dT(x)}{dx} = 0 \quad \text{when} \quad x = H$$

From these $A = T_p(0) \tanh\beta - \dfrac{1}{\alpha\cosh\beta}\dfrac{dT_p(x)}{dx}$ (3.21.a)

$$B = - T_p(0)$$

where $T_p(0)$ = the value of particular integral, Eq.(3.20), when x is zero

and $\beta = \alpha H$

3.4.5.2 Another common situation arises when two shear walls are interconnected by an "infinitely" rigid diaphragm at the top of the structure. To this reference was made in Chapter 2. In that case the boundary conditions may be defined as follows:

a.) Because of the presence of an infinitely rigid diaphragm at the top of the structure, the topmost lamina cannot deform. Thus no load is induced in the same and

$$q(x) = \frac{dT(x)}{dx} = 0 \quad \text{when} \quad x = 0$$

b.) The boundary conditions at the base are the same as in the previous case, 3.4.5.1.b. From these

$$A = -\frac{dT_p(0)}{\alpha dx} \qquad (3.22.a)$$

$$B = \frac{dT_p(0)}{\alpha dx} \frac{1}{\tanh\beta} - \frac{1}{\sinh\beta} \frac{dT_p(H)}{\alpha dx} \qquad (3.22.b)$$

3.4.6 The External Load

Two load patterns only, commonly representing seismic conditions, are considered here.

3.4.6.1 A distributed triangular lateral load is acting with a maximum intensity at the top of the structure. The moment intensity is

$$M_o = WH(\xi^2 - \frac{\xi^3}{3}) \qquad (3.23)$$

where W = the total triangular lateral load, also referred to as base shear in seismic load analysis.

$\xi = \frac{x}{H}$; hence $0 \leqslant \xi \leqslant 1$

For this load, only the first two terms of the particular integral are non-zero. Therefore

$$T_p(\xi) = \frac{\gamma WH^3}{\beta^2}[\xi^2 - \frac{\xi^3}{3} + \frac{2}{\beta^2}(1-\xi)] \text{ and } T_p(0) = \frac{2\gamma WH^3}{\beta^4}$$

$$\frac{dT_p(0)}{dx} = -\frac{2\gamma WH^2}{\beta^4} \text{ and } \frac{dT_p(H)}{dx} = \frac{\gamma WH^2}{\beta^2}(1 - \frac{2}{\beta^2})$$

3.4.6.2 A single point load, P, applied laterally at the top of the structure gives the following quantities

$$M_o = PH\xi \qquad (3.24)$$

$$T_p(\xi) = \frac{\gamma PH^3}{\beta^2}\xi \text{ and } \frac{dT(0)}{dx} = \frac{dT(H)}{dx} = \frac{\gamma PH^2}{\beta}$$

3.4.7 Coupled Shear Walls Subject to Lateral Triangular and Single Point Loads

After substituting the appropriate values for the particular integral, its derivatives, and the integration constants into Eqs. (3.21.a) and (3.21.b) the unknown actions can be obtained from the fundamental differential equation (3.19) as follows:

$$T(\xi) = \frac{\gamma W H^3}{\beta^2}\Big[\frac{2}{\beta^2}\tanh\beta\sinh\beta\xi + \frac{\sinh\beta\xi}{\cosh\beta}\Big(\frac{2}{\beta^3} - \frac{1}{\beta} - \frac{\rho}{\beta}\Big) - \\ - \frac{2}{\beta^2}\cosh\beta\xi - \frac{\xi^3}{3} + \xi^2 + \frac{2}{\beta^2}(1-\xi) + \rho\xi\Big] \qquad (3.25)$$

Where $\rho = \frac{P}{W}$ is the load ratio. This is assumed to remain constant throughout the load history of the structure.

$$q(\xi) = \frac{dT(\xi)}{Hd\xi} = \frac{\gamma W H^2}{\beta}\Big[\frac{2}{\beta^2}\tanh\beta\cosh\beta\xi + \frac{\cosh\beta\xi}{\cosh\beta}\Big(\frac{2}{\beta^3} - \frac{1}{\beta} - \frac{\rho}{\beta}\Big) - \\ - \frac{2}{\beta^2}\sinh\beta\xi + \frac{1}{\beta}(2\xi - \xi^2) - \frac{2}{\beta^3} + \frac{\rho}{\beta}\Big] \qquad (3.26)$$

The shearing forces generated by the separation forces are obtained from Eq. (3.8) thus

$$V_p = \frac{dM_p}{dx} = C\frac{dT(x)}{dx} = Cq(x) \qquad (3.27)$$

Hence the separation forces are in general

$$p(x) = \frac{dV_p}{dx} = C\frac{dq(x)}{dx} \qquad (3.28)$$

or for the particular shear wall structure under consideration

$$p(\xi) = \gamma HCW\Big[\frac{2}{\beta^2}\tanh\beta\sinh\beta\xi + \frac{\sinh\beta\xi}{\cosh\beta}\Big(\frac{2}{\beta^3} - \frac{1}{\beta} - \frac{\rho}{\beta}\Big) - \\ - \frac{2}{\beta^2}\cosh\beta\xi + \frac{2}{\beta^2}(1-\xi)\Big] \qquad (3.29)$$

where C was given by Eq. (3.7).

If these separation forces are defined by a continuously distributed load pattern then the shear force induced by these at the top of the structure should be zero. However, it is seen from Eq. (3.27) that

$$V_p(0) = Cq(0) \neq 0 \text{ unless } C = 0$$

which is generally the case of structural symmetry. For this reason at the topmost lamina a singular separation force, P_o, - to the writer's knowledge not shown in any of the previous studies, - must also exist. This is given by

$$P_o = Cq(0) = \gamma H^2 CW \left[\frac{1}{\cosh\beta} \left(\frac{2}{\beta^2} - \rho - 1 \right) + \rho - \frac{2}{\beta^2} \right] \quad (3.30)$$

To be able to estimate the fundamental period of the structures its deflection under a prescribed load must be known. For the previously defined triangular and point load the lateral deflection of the shear wall is easily found from the familiar relationship

$$\frac{dy^2}{dx^2} = \frac{1}{EI_o} \left[M_o - lT(x) \right]$$

By integrating twice and by observing the boundary requirements, the deflection of the structure at any level is obtained from

$$\begin{aligned} y = \frac{WH^3}{EI_o} \Big\{ & \frac{1}{60}\left(\frac{\gamma l}{\alpha^2} - 1\right)\left[\xi^5 - 5\xi^4 + 15\xi - 11 - 10\rho(\xi^3 - 3\xi + 2)\right] - \\ & - \frac{\gamma l}{\alpha^2 \beta^4 \cosh\beta}\Big[(\sinh\beta\xi - \beta\xi\cosh\beta - \sinh\beta + \beta\cosh\beta)\left(2\sinh\beta + \frac{2}{\beta} - \beta - \beta\rho\right) - \\ & - 2\cosh\beta(\cosh\beta\xi - \beta\xi\sinh\beta - \cosh\beta + \beta\sinh\beta) - \\ & - (\xi^3 - 3\xi^2 + 3\xi - 1)\frac{\beta^2\cosh\beta}{3}\Big]\Big\} \end{aligned} \quad (3.31)$$

The deflection at the top of the structure is

$$\begin{aligned} y_o = \frac{WH^3}{EI_o} \Big\{ & \left(1 - \frac{\gamma l}{\alpha^2}\right)\left(\frac{11}{60} + \frac{\rho}{3}\right) - \frac{\gamma l}{\alpha^2\beta^4}\Big[(\beta - \tanh\beta)\left(2\sinh\beta + \frac{2}{\beta} - \beta - \rho\beta\right) - \\ & - 2(1 - \cosh\beta + \beta\sinh\beta) + \frac{\beta^2}{3}\Big]\Big\} \end{aligned} \quad (3.32)$$

It may be easily shown that, if the coupling beams are very stiff (i.e. I_x is very large), the equation for the laminar shear, Eq. (3.26) reduces to

$$q(\xi) = \frac{1}{I_o\left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{l^2}{I_o}\right)} \left[W\xi(2 - \xi) + P\right]$$

This is identical with Jourawski's shear equation, applicable to a beam in which the Bernoulli-Navier strain distribution holds.

The typical distributions of the laminar quantities are shown in Fig. 3.5.

3.4.8 Coupled Shear Wall With a Rigid Diaphragm at the Top

In 3.4.5.2 the requirements satisfying the boundary conditions for two coupled shear walls, which are interconnected by an infinitely rigid diaphragm at roof level, have been established. Hence by substituting the appropriate values of the integration constants, Eqs. (3.22.a) and (3.22.b) into the general solution, Eq. (3.19), the laminar quantities are obtained thus:

$$T(\xi) = \frac{\gamma W H^3}{\beta^2}\left[\left(\frac{2}{\beta^3} - \frac{\rho}{\beta}\right)\left(\sinh\beta\xi + \frac{\cosh\beta\xi}{\sinh\beta} - \frac{\cosh\beta\xi}{\tanh\beta}\right) - \right.$$
$$\left. - \frac{\cosh\beta\xi}{\beta\sinh\beta} - \frac{\xi^3}{3} + \xi^2 + \frac{2}{\beta}(1 - \xi) + \rho\xi\right] \qquad (3.33)$$

$$q(\xi) = \frac{\gamma W H^2}{\beta}\left[\left(\frac{2}{\beta^3} - \frac{\rho}{\beta}\right)\left(\cosh\beta\xi + \frac{\sinh\beta\xi}{\sinh\beta} - \frac{\sinh\beta\xi}{\tanh\beta}\right) - \right.$$
$$\left. - \frac{\sinh\beta\xi}{\beta\sinh\beta} + \frac{1}{\beta}(2\xi - \xi^2) - \frac{2}{\beta^3} + \frac{\rho}{\beta}\right] \qquad (3.34)$$

$$p(\xi) = \gamma HCW \left[\left(\frac{2}{\beta^3} - \frac{\rho}{\beta}\right)\left(\sinh\beta\xi + \frac{\cosh\beta\xi}{\sinh\beta} - \frac{\cosh\beta\xi}{\tanh\beta}\right) - \right.$$

$$\left. - \frac{\cosh\beta\xi}{\beta\sinh\beta} + \frac{2}{\beta^2}(1 - \xi)\right] \qquad (3.35)$$

With reference to Eq.(3.30) it may be observed that the normally generated singular separation force at the topmost lamina becomes zero because the laminar shear intensity at this level is also zero. (See 3.4.5.2.a). However in the rigid diaphragm itself considerable shear may be induced. This shearing force is transmitted as axial tension or compression directly to the top of the walls. Therefore

the shear force in the diaphragm = T(0)

$$\text{where} \quad T(0) = \frac{\gamma WH^3}{\beta^2}\left[\frac{1 - \cosh\beta}{\sinh\beta}\left(\frac{2}{\beta^3} - \frac{\rho}{\beta}\right) - \frac{1}{\beta\sinh\beta} + \frac{2}{\beta}\right] \qquad (3.36)$$

This may be seen to increase with the decrease of $\beta = \alpha H$ i.e. the stiffness of the coupling beams. (Eq. 3.16).

The lateral deflection of this structure may be derived in a similar way to that shown for the other boundary conditions by Eqs. (3.31) and (3.32). It is found that

$$y = \frac{WH^3}{EI_o}\left\{\frac{1}{60}\left(\frac{\gamma 1}{\alpha^2} - 1\right)\left[\xi^5 - 5\xi^4 + 15\xi - 11 - 10\rho(\xi^3 - 3\xi + 2)\right] - \right.$$

$$- \frac{\gamma 1}{\alpha^2\beta^4}\left[\left(\frac{2}{\beta} + \rho\beta\right)(\sinh\beta\xi - \sinh\beta + \beta - \beta\xi) + \right.$$

$$+ \left(\frac{2}{\beta} + \rho\beta\right)\left(\frac{\cosh\beta\xi - \cos\beta}{\sinh\beta}\right)(1 - \cosh\beta) +$$

$$\left.\left. + \beta\frac{(\cosh\beta - \cosh\beta\xi)}{\sinh\beta} - \frac{\beta^2}{3}(\xi^3 - 6\xi + 5)\right]\right\} \qquad (3.37)$$

and that the deflection at the top of the walls is

$$y_o = \frac{WH^3}{EI_o} \{(1 - \frac{\gamma 1}{\alpha^2})(\frac{11}{60} + \frac{\rho}{3}) - \frac{\gamma 1}{\alpha^2 \beta^2} [(\frac{2}{\beta^2} - \rho)(1 + \frac{2 - 2\cosh \beta}{\beta \sinh \beta}) +$$

$$+ \frac{\cosh \beta - 1}{\beta \sinh \beta} - \frac{5}{3}]\} \qquad (3.38)$$

The typical distribution of the laminar shear and the bending moment for wall 2 are shown also for this case in Fig. 3.5 by dotted lines. From these it may be seen that the lower parts of the walls, which usually become critical in design, are not affected by the type of connection which may exist at the top of the structure.

3.5 The Application of the Laminar Analysis to the Real Structure

Once the laminar analysis has been completed it becomes necessary to transform the laminar quantities into discrete quantities so that the coupling beams and the walls can be designed. From most published studies one may get the impression that the laminar analysis is directly transferable to the structure. The errors, which would result from such a procedure, may be quite considerable particularly for buildings of moderate heights.

One coupling beam was replaced by a number of laminae, which were spread out over the height of one storey, h . The accumulation of actions in these laminae must be resisted by a discrete coupling beam. Thus the shear force across one beam situated at $x = x_1 + \frac{h}{2}$ is

$$V = \int_{x_1}^{x_1 + h} q(x)dx$$

where x_1 = the ordinate of any point situated half-way between the centre lines of coupling beams.

The maximum moment at the support of the beam is

$$M_{max} = \frac{Vs}{2}$$

and the accumulated axial force, resulting from the separation forces is

$$P_b = \int_{x_1}^{x_1+h} p(x)\,dx$$

The shear, axial force and bending moment induced in the walls can be determined directly from the laminar analysis, but only at points defined by the x_1 ordinates. The summation of the laminar quantities above these points yields the same results as the summation of the discrete quantities. However between these points, i.e. at the centre lines of the coupling beams, discontinuities occur which must normally be determined. The previously obtained continuous curves, such as shown in Fig. 3.5, must be adjusted accordingly[52].

3.6 A Critical Examination of the Assumptions of the Laminar Analysis

It is appropriate to be reminded that the subjects of this study are reinforced concrete coupled shear wall structures. The prime purpose of such a structure is to resist relatively large lateral forces generated by wind pressure or seismic accelerations. In the process of disposing of such forces, through its foundations, several or all major components of the structure may crack. This perfectly natural phenomenon may considerably alter the behaviour of the structure. Therefore the likely performance of shear walls in the uncracked and cracked state are examined a little closer.

3.6.1 Uncracked Coupled Shear Walls

The eleven previously (in 3.2) made assumptions are generally satisfied in the uncracked concrete structure to such an extent that very satisfactory results can be expected for design purposes. Photoelastic studies have verified this. However a few exceptions may warrant a brief examination.

3.6.1.1 Assumption "b"

Coupling beams, particularly in office buildings, are frequently deep in comparison to their span. The flexural strains and stresses are no longer distributed in a linear fashion. With a suitable Airy stress function, finite difference or finite element analysis or by means of photoelastic models, the actual stress distribution can be determined. To the writer's knowledge only two photoelastic studies were carried out for the load pattern which is typical for coupling beams[14,38]. The smaller is the aspect ratio (s/D) of a beam the larger is the deviation of the flexural stresses from the linear pattern at the support. However towards the midspan a near linear stress distribution is rapidly developed.[38]

From these observations it may be concluded that the assessment of beam stiffness may be carried out with sufficient accuracy using the conventional flexural theory only.

3.6.1.2 Assumption "e"

Sectional properties of walls often vary with the height because the wall thickness can be conveniently decreased toward the top of the structure. It is also common to change the thickness of the coupling beams so as to match the thickness of the walls. The sectional properties of walls and beams are, usually with good approximation, linearly proportional to the thickness. The ratio of their properties, as expressed by the parameters γ and α (Eqs. (3.16) and (3.17)) does not significantly change. For this reason it is not expected that changes in the load distribution between walls and coupling beams are worth considering, when the thickness of the components is reduced with the height of the building.

3.6.1.3 Assumption "g"

This assumption is made to enable the laminar replacement of the coupling beams at the top of the structure to be satisfied. It was already shown in 3.4.8 that the critical parts of the structure do not react to major changes which occur at the top of the structure. The most serious violation of this assumption has little effect upon the critical design quantities.

3.6.1.4 Assumption "h"

In most structures this assumption is quite acceptable because the axial forces, such as may occur from the accumulation of separation forces are very small. More often a floor slab, which frames into the coupling beam, provides further restraint against extensional deformations.

3.6.1.5 Assumption "i"

The common type of coupled shear wall structures, in which critical conditions can arise, usually possess a large height to depth ratio. For this reason the shear deformations represent a minor fraction of the total deformations, and thus the effect of shear upon the stiffness of the walls is usually insignificant.

3.6.1.6 The limitations of model studies

Reinforced concrete coupled shear wall structures can remain crack-free only if the lateral load is very small. Any analysis, which aims to predict the critical tensile stresses in the concrete components, has limited application in countries prone to earthquakes. Similarly photoelastic studies, which reveal stress concentrations, are of little use in predicting the behaviour and strength of shear wall structures. Stress concentrations, such as occur at the corners of openings, vanish after the formation of a few minor cracks and do not affect the behaviour of the real structure. This will also be evident from experiments reported in later chapters.

3.6.2 Cracked Coupled Shear Walls

Cracking of the concrete, which manifests that the reinforcement has commenced to contribute more effectively to the strength of the structure, may significantly alter the behaviour of coupled shear walls. This becomes more evident when the effect of cracking upon the components of the structure is examined in greater detail.

3.6.2.1 The cracking of walls

The cracking of a cantilever wall is likely to commence at its base. Consequently there is a loss of flexural rigidity over the affected height, and this leads to increased flexural rotations. These affect the rotations of every other part of the upper stories of the structure, including the coupling beams.

The onset of cracking in the walls is also affected by the axial forces present. By adding the moment equations for each of the two walls Eqs. (3.9.a) and (3.9.b), the overall equilibrium statement can be formulated thus:

$$M_o = M_1 + M_2 + lT(x) \tag{3.39}$$

This states that the external moment is resisted by an internal moment in each of the two walls, M_1 and M_2, and by a force couple $T(x)$ operating on a lever arm l. Hence axial forces must always be present unless the coupling beams are rendered ineffective. In one of the walls axial tension is induced and this will promote the development of cracks. In the other wall the formation of cracks may be considerably delayed or supressed by axial compression.

In a well balanced design considerable gravity stresses are imposed by the dead and live load. These compression stresses delay the formation of flexural cracks.

It is thus evident that the formation and development of cracks in the walls, and the consequent loss of stiffness, is greatly affected by the intensity of axial load present. After cracking a geometrically symmetrical reinforced concrete shear wall structure becomes structurally non-symmetrical. In general it may be expected that, because of their greater absolute strength and the beneficial effect of gravity loads, cracking in the walls occurs after the majority of the coupling beams have cracked.

3.6.2.2 The cracking of coupling beams

The coupling beams are subject to flexure, in the presence of large shearing forces. Because of stress concentrations, cracking in such beams may be expected to occur at a load less than what a conventional flexural stress analysis, based on the tensile strength of the concrete, would predict. Because the shear stresses are frequently large, flexural cracks rapidly incline or new diagonal cracks form. It will be shown later that the relative proportions of shear deformations in diagonally cracked beams can be considerably greater than the corresponding proportions in cracked beams. Shear deformations may well exceed the simultaneously occurring flexural deformations.

There is some evidence which suggests that the coupling beams may be the weakest elements in a coupled shear wall structure. In two similar 14 storey buildings most of the coupling beams failed completely in the 1964 Alaska earthquake[56]. See Fig. 1.1, Fig. 1.2, Fig. 1.3 and Fig. 1.4. One of the walls failed due to axial tension near its base, but the remainder of the walls appear to have remained undamaged.

The relatively early onset of cracking in the coupling beams must result in a loss of laminar shear transfer and a consequent loss of axial force induced in the walls. Hence, according to Eq. (3.39), the wall moments must increase.

Clearly in a cracked structure many of the previously made assumptions are violated. The effect of cracking in a coupled shear wall structure is likely to be more significant because its components, the walls and the coupling beams, are affected to very different extents by the loss of stiffness.

3.7 The Approximate Analysis of Cracked Coupled Shear Wall Structures

From the foregoing discussion on the effects of cracking, it is evident that the laminar analysis, carried out in terms of stresses[50,51], has limited application. The structural designer is more interested in static quantities which enable him to proportion and detail the components so that they can efficiently resist these actions.

To enable the static quantities to be derived from a laminar analysis which takes into account, at least in an approximate way, the effect of cracking, it is necessary to assume uniform properties separately for each of the three main components of the coupled shear wall structure. A preliminary analysis, based on uncracked sections, will indicate the likely intensity of actions, from which the extent of cracking may be estimated.

For a second analysis it may be assumed that the stiffness of all coupling beams is reduced by cracking. A reduction of laminar shear and an increase of wall moments will be indicated by the analysis. A considerable amount of experimental evidence is offered in subsequent chapters to clarify the elastic behaviour of cracked coupling beams.

In a third analysis the loss of stiffness of the wall may be considered. The extent of cracking owing to flexure and axial tension is likely to be restricted to the lower portions of the wall. The assumption that the stiffness reduces uniformly over the full height of the wall may be a crude one. However it will yield conservative static quantities for the other wall and for the coupling beams.

For the final design the most critical situation may be considered at any section. In the absence of more accurate data the stiffnesses may be varied between limits, according to the judgement of the designer. Clearly this will lead to conservative envelopes within which a number of statically admissible situations of the elastic behaviour are contained.

A numerical example showing the application of the proposed approximate procedure is given in the next section.

3.8 Illustrative Example

To illustrate the effect of stiffness loss subsequent to cracking, an 18 storey shear wall core will be considered. Openings at each floor pierce the opposite walls of a uniform box section. Reference may be made to Fig. 3.1 with respect to notation. The properties of the structure and the loading are summarised in Table 3.I below.

TABLE 3.I PROPERTIES OF 18 STOREY COUPLED SHEAR WALLS

LOADING CASE	WALL PROPERTIES				BEAM* AREA	NOTES
	A_1	A_2	I_1	I_2		
	$\times 10^3$ in.^{2}	$\times 10^3$ in.^{2}	$\times 10^6$ in.^{4}	$\times 10^6$ in.^{4}	in.^{2}	
Case A	5.55	5.65	6.60	9.00	672	Uncracked
Case B	5.55	5.65	6.60	9.00	202	Beam cracked
Case C	3.90	5.65	3.30	9.00	202	Beam and Wall 1 cracked

* For two beams

Constant Data:

Floor height h = 105 in. Total height H = 1890 in.

Spans: l = 168.7 in., l_1 = 82.2 in., l_2 = 86.5 in.

Beam depth D = 24 in., width b = 14 in., span s = 38 in.

$E = 5 \times 10^6$ p.s.i.

Triangular load W = 500 Kips, Point load P = 65 Kips.

In order to study the effect of cracking three cases are considered as follows:

Case A: It is assumed that the entire structure remains crack free during the loading.

Case B: It is assumed that all coupling beams crack and that, as a consequence of this, the stiffness of the beams is reduced by 70%. The beam area is reduced in the analysis accordingly.

Case C: Wall 1, situated at the tension side of the structure has also cracked so that its equivalent area is reduced by 30%. The corresponding reduction of the second moment of area is 50%.

The results of the analysis are presented in Fig. 3.5. The diagrams show that

a.) cracking had little effect on the axial forces, T, generated in the walls,

b.) the maximum shear force across one coupling beam at about the 4th floor is approximately

$$V = .5 \times 105 \times 2.55 \doteq 134 \text{ Kips}$$

which corresponds to a nominal shear stress of approximately $v = \frac{134000}{14 \times 21} = 450$ p.s.i.

The same shear force would induce a maximum beam moment at the supports of

$$M_B = 0.5 \times 38 \times 134 = 2550 \text{ Kip in.}$$

Clearly no reinforced concrete beam of the given dimensions would be capable of carrying such shear and moment simultaneously without cracking.

c.) The maximum bending moment in Wall 2, which is subject to axial compression, increased by 50% as a result of cracking in the remainder of the structure.

CHAPTER FOUR

AN ULTIMATE LOAD ANALYSIS OF COUPLED SHEAR WALLS

4.1 Introduction

The ultimate strength of two coupled shear walls, subject to seismic type of lateral loading, is obtained when a statically admissible mechanism is formed in which each of the required plastic hinges possesses the required rotational capacity. Two hinges in each coupling beam are required to terminate its ability to accept further shear. In addition one plastic hinge need be developed in each of the cantilever walls, normally at their base, to complete the collapse mechanism. The sequence of hinge formation for a given loading will depend upon the relative strength and stiffness of the components.

The behaviour of some shear walls[27,56], which were exposed to severe earthquakes, indicated that all or most coupling beams failed before the ultimate strength of the coupled walls was attained. However it is possible that in some structures the ultimate strength of the walls need be exhausted before plastic hinges can form in the coupling beams.

Coupled shear wall structures frequently consists of members which have unusual relative dimensions. Therefore it can be expected that the techniques of current ultimate load analyses, which consider relatively slender members only, may not be fully applicable. Before a satisfactory ultimate load approach can be established for the design of coupled shear wall structures, it is essential to investigate the behaviour of its components.

In order to establish the principles of overall behaviour a brief analytical study is presented, in which the important stages of the formation of the collapse mechanism are examined. To do this a sequence of hinge-formation need be assumed. After establishing the equilibrium requirements the compatibility conditions are studied so as to reveal the rotational requirements at the plastic hinges.

The type of structure which is examined consist of two coupled shear walls and a coupling system, the strength of which is relatively small. Therefore it is expected that plastic hinges form first in each of the confining beams before plastic deformations of any kind would occur in the walls.

In the absence of experimental information on the interaction of flexure shear and axial tension, it is considered unwise to proceed further, and to consider failure mechanisms in which plastic hinges also form at the base of the walls. From reports on structural damage[27,56] caused by earthquakes, the writer was unable to identify this type of failure mechanism, except in structures where the wall dimensions were drastically reduced at foundation level or where columns were used to support coupled shear walls.

4.2 The Maximum Load Carried by the Elastic Structure

According to the assumptions previously stated the ultimate strength of the coupling beams is attained first, as the lateral load is gradually increased. The elastic limit of the structure is that load, W_e, at which the most highly loaded coupling beam or lamina is about to yield. For the purpose of this study it is assumed that beams and walls behave according to a bilinear elasto-plastic moment-rotation relationship.

In order to determine the above defined elastic limit, W_e, it is necessary to locate the most highly loaded elastic lamina. Differentiation of Eq.(3.26) leading to Eq.(3.28) will yield this condition, i.e.

$$\frac{dq(x)}{dx} = \frac{1}{C}p(x) = 0 \tag{4.1}$$

from which the position $x_e = \xi_e H$, at which the laminar shear attains its maximum value

$$q(\xi_e) = q_{max} \tag{4.2}$$

can be determined. This can be most conveniently obtained from a plot of the $q(\xi)$ or $p(\xi)$ functions, i.e. Eq. (3.26) and Eq. (3.29) or from a trial and error solution of the latter equation in this form:

$$\sinh \beta\xi = \frac{\beta^5 \cosh \beta}{\beta^5 \sinh \beta - 2\beta^2(1+\rho)+2}(\xi - 1) \tag{4.3}$$

Having obtained the value of the maximum laminar shear, q_{max}, the load, at which the ultimate shear capacity of this particular lamina is reached, is found by proportion from

$$W_e = \frac{q_u}{q_{max}} W \tag{4.4}$$

where W_e = is the maximum lateral load carried by the fully elastic structure

q_u = the maximum laminar shear which is to be determined from the ultimate load capacity, V_u, of the coupling beams.

Similarly the deflections due to the maximum elastic load may be found by proportion

$$y_e = \frac{W_e}{W} y \quad \text{and} \quad y_{oe} = \frac{W_e}{W} y_o \tag{4.5}$$

from Eqs. (3.31) and (3.32).

4.3 The Plastification of the Laminar System

In the next stage of this study the situation is examined in which the coupling system has attained its ultimate strength while the coupled walls still operate in the linear elastic range. Because the characteristics of all laminae are the same after yielding occurred in the last one, it is possible to obtain an approximation - good enough for design purposes - by slightly modifying the previously presented laminar analysis.

4.3.1 Actions

Because the laminar shear, q_u , is constant at this stage of the loading, the induced axial force, Eq.(3.2) becomes

$$T(x) = \int_o^x q_u dx = q_u x \tag{4.6}$$

Consequently the moment equations, Eq.(3.9), simplify to

$$M_1 = \frac{I_1}{I_o}(M_o - 1q_u x) \tag{4.7.a}$$

$$M_2 = \frac{I_2}{I_o}(M_o - 1q_u x) \tag{4.7.b}$$

and from Eq. (3.8) $M_p = Cq_u x$ (4.8)

The shearing force induced by the separation forces is

$$V_p = \frac{dM_p}{dx} = Cq_u \tag{4.9}$$

constant, indicating that only a single separation force

$$P_o = V_p$$

is required to act at the top so as to ensure equal deformation of the walls.

4.3.2 Rotations

It is also necessary to examine the rotations which occur in the laminae in the process of developing the ultimate laminar shear q_u. It was shown in 3.4.4 that the compatibility of wall and laminar deformations is satisfied when

$$d_m = d_b + d_a \tag{4.10}$$

where d_m and d_a were defined by Eq. (3.10) and Eq. (3.4). These were also indicated in Fig. 3.4. The laminar displacement resulting from flexural and shear distortions was determined from Eq. (3.14). However, because of the plastic behaviour of the laminae, it is more convenient to express this in terms of beam rotations thus,

$$d_b = s\theta_b = s(\theta_y + \theta_p) \tag{4.11}$$

where θ_b = the total laminar rotation consisting of two parts.
θ_y = the maximum elastic rotation of the laminae at the onset of yielding. This is the same for all laminae.
θ_p = the plastic laminar rotation required to occur after the onset of yielding. This varies along the height of the structure.

The elastic wall rotations, θ_w, may be expressed in terms of d_m i.e.

$$\theta_w = \frac{d_m}{l}$$

The wall and laminar rotations are compatible, Eq. (4.10) and Eq. (4.11) when

$$l\theta_w = s(\theta_y + \theta_p) + d_a = l\frac{dy_1}{dx} = l\frac{dy_2}{dx} = l\frac{dy}{dx}$$

Hence the plastic laminar rotations, which need be developed when the ultimate laminar shear is attained in all laminae, are found from

$$\theta_p = \frac{l}{s}\theta_w - \frac{da}{s} - \theta_y \tag{4.12}$$

Clearly it is not possible to develop yield rotation in the first laminae adjacent to the base because the elastic cantilever walls are fully restrained against rotations at this level. Thus it is not possible to develop the ultimate laminar shear, q_u, over the entire height of the structure. However, if the yield rotation, θ_y, in the laminae can occur close enough to the base a satisfactory approximation may be made. One may stipulate that the laminar rotations should equal or exceed the yield rotation over at least 90% of the height of the coupling system. In such a case the maximum axial force generated in the walls is

$$T_{max} > .95 q_u H$$

The error involved, in assessing the contribution of the laminar forces towards the ultimate strength of the structure, is less than 5%.

By combining Eq. (3.10) and Eq. (4.6) for θ_w and also using Eq. (3.4) for d_a it is found that

$$\theta_p = \frac{1}{sEI_o} \int_x^H (M_o - 1q_u x)dx - \frac{1}{sE}\left(\frac{1}{A_1} + \frac{1}{A_2}\right)\int_x^H 1q_u x dx - \theta_y \qquad (4.13)$$

When the appropriate moment expressions from Eqs. (3.23) and (3.24) are used for the specified load pattern the plastic laminar rotations become

$$\theta_p = \frac{1H^2}{sEI_o}\left\{ W\left[\frac{1}{4} + \frac{\xi^4}{12} - \frac{\xi^3}{3} + \frac{\rho}{2}(1 - \xi^2)\right] - \frac{1q_u}{2}(1 - \xi^2)\right\} - \frac{q_u H^2}{2sE}\left(\frac{1}{A_1} + \frac{1}{A_2}\right)(1 - \xi^2) - \theta_y \qquad (4.14)$$

It may now be specified that the yield rotation be just attained in a specific lamina, i.e.

$\theta_b = \theta_y$ thus from Eq. (4.11) $\theta_p = 0$ when $\xi = .9$

This will occur under the action of one specific load, W_y, which is found from Eq.(4.14) thus

$$W_y = \frac{sEI_o(0.19Z + 1)}{lH^2(0.0616 + 0.095\rho)} \theta_y \tag{4.15}$$

where $$Z = \frac{q_u H^2}{2sE\theta_y}\left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{l^2}{I_o}\right) \tag{4.16}$$

Having obtained the load at which 90% of the laminae attained or exceeded the yield rotation the distribution of postelastic rotations can be determined from Eq. (4.14) and (4.15) conveniently in this form,

$$\frac{\theta_p}{\theta_y} = \frac{1 + 0.19Z}{0.0616 + 0.095\rho}\left[\frac{1}{4} + \frac{\xi^4}{12} - \frac{\xi^3}{3} + \frac{\rho}{2}(1 - \xi^2)\right] - Z(1 - \xi^2) - 1 \tag{4.17}$$

The particular features of this expression are that:

a.) $\frac{\theta_p}{\theta_y} = 0$ when $\xi = .9$ showing that no plastic rotations occur at the specified level and

b.) $\frac{\theta_p}{\theta_y} = -1$ when $\xi = 1.0$ indicating that a negative plastic rotation is required in the bottom lamina to ensure that the total rotation there is zero. This absurd situation results from the incompatibility of the boundary conditions at the base and the assumption that a uniform laminar shear, q_u, is induced over the entire height. The consequent small error was discussed previously.

It was stated that the maximum laminar shear, q_u, is attained over 90% of the height of the structure only if plastic laminar rotations occur. The satisfying of this requirement must be examined.

The laminar rotations generally reduce towards the top of the structure because the moments, generated by the laminar forces, tend to deflect the walls in the direction opposite to the external load. Negative plastic rotations, which Eq. (4.17) may yield, would indicate that the yield rotation in a particular lamina has not been attained. Plastification of the laminae over 90% of the height occurs only if

B $$\frac{\theta_p}{\theta_y} \geq 0 \quad \text{also when} \quad \xi = 0$$

Hence from Eq. (4.17)

$$\frac{1 + 0.19Z}{0.0616 + 0.095\,\rho}\left(\frac{1}{4} + \frac{\rho}{2}\right) - (Z + 1) \geq 0$$

from which

$$Z \leq \frac{\nu - 1}{1 - 0.19\nu} = Z_{critical} \tag{4.18}$$

where $$\nu = \frac{0.25 + 0.50\,\rho}{0.0616 + 0.095\rho} \tag{4.19}$$

Thus when Z, a stiffness ratio, is large the yield rotations in the upper laminae are not attained. Consequently the assumptions of the above approach are not met. From Eq. (4.16) it is evident that this may be the case when relatively flexible walls are coupled by stiff laminar beams.

When $Z \ngtr Z_{critical}$ it is necessary to stipulate that the yield rotation be attained at the topmost lamina instead of the one situated near the base, i.e. when $\xi = .9$. By specifying that

$$\theta_b = \theta_y \quad \text{thus} \quad \theta_p = 0 \quad \text{when} \quad \xi = 0$$

and by expanding Eq. (4.14) the specific value of the load, W'_y, responsible for this situation can be derived in a similar way to that shown in Eq. (4.15). It is found that

$$W_y' = \frac{4_s E I_o (Z+1)}{1H^2 (1+2\rho)} \theta_y \qquad (4.15.a)$$

The postelastic rotations are found from Eqs. (4.14) and (4.15.a) in a form which is similar to Eq. (4.17), thus

$$\frac{\theta_p}{\theta_y} = \frac{4(Z+1)}{1+2\rho} \left[\frac{1}{4} + \frac{\xi^4}{12} - \frac{\xi^3}{3} + \frac{\rho}{2}(1-\xi^2) \right] - Z(1-\xi^2) - 1 \qquad (4.17.a)$$

The features of this equation are:

a.) $\frac{\theta_p}{\theta_y} = 0$ when $\xi = 0$

b.) $\frac{\theta_p}{\theta_y} \geq 0$ when $\xi = .9$ because $Z \geq Z_{critical}$

This shows that the plastification extends in excess of 90% of the structure's height.

c.) $\frac{\theta_p}{\theta_y} = -1$ when $\xi = 1.0$ because of the incompatibility of rotations at the base.

Eqs. (4.17) and 4.17.a) are important ones, for they enable the rotational requirements for each of the laminae to be compared with the available rotational capacity.

The example given in 4.6 illustrates cases on both sides of the critical value of the stiffness ratio Z.

4.3.3 Deflections

It was stipulated that the walls remain elastic during the previously described process of laminar plastification. Consequently the wall deformations can be readily obtained when the structure is subject to the load W_y or W_y' which induced the plastification of the laminae over at least 90% of the height. This wall deflection is given by

$$y_{py} = \frac{H^3}{EI_o} \left\{ W_y \left[\frac{11}{60} - \frac{\xi^5}{60} + \frac{\xi^4}{12} - \frac{\xi}{4} + \frac{\rho}{2}\left(\frac{\xi^3}{3} - \xi + \frac{2}{3}\right) \right] - \frac{1q_u}{2}\left(\frac{\xi^3}{3} - \xi + \frac{2}{3}\right) \right\} \qquad (4.20)$$

The maximum deflection at the top of the structures is

$$y_{opy} = \frac{H^3}{EI_o}\left[W_y\left(\frac{11}{60} + \frac{\rho}{3}\right) - \frac{1q_u}{3}\right] \tag{4.21}$$

The lateral deflections increase from y at the maximum elastic load, Eq. (4.5) to y_{py} when under the action of W_y (Eq. (4.15)) or W'_y (Eq. (4.15.a)).

4.4 The Ultimate Load

According to the assumed sequence of plastification it is possible to further increase the load on the structure provided that the plastified laminae are capable of maintaining the ultimate shear during subsequent rotations. Any additional load need be carried entirely by cantilever action of the coupled walls.

4.4.1 The Ultimate Load on Wall 1

If the additional load, required to cause the moment at the base of Wall 1 to reach its ultimate value, is $\Delta W'$, then the moments induced in both walls are

$$M_1(\xi) = \frac{HI_1}{I_o}\left[(W_y + \Delta W')\left(\xi^2 - \frac{\xi^3}{3} + \rho\xi\right) - 1q_u\xi\right] \tag{4.22.a}$$

$$M_2(\xi) = \frac{I_2}{I_1} M_1(\xi) \tag{4.22.b}$$

the critical moments at the base of the walls become

$$M_1(1) \leq \frac{HI_1}{I_o}\left[(W_y + \Delta W')\left(\frac{2}{3} + \rho\right) - .95\,1q_u\right] = M_{1,u} \tag{4.23}$$

$$M_2(1) = \frac{I_2}{I_1} M_{1,u} \leq M_{2,u} \tag{4.24}$$

where $M_{1,u}$ and $M_{2,u}$ = the ultimate moment capacities of the walls at the base.

The elastic wall rotations due to the additional load $\Delta W'$ follow from the elastic cantilever actions only and are

$$\theta_w' = \frac{\Delta WH^2}{EI_o}\left[\frac{1}{4} - \frac{\xi^3}{3} + \frac{\xi^4}{12} + \frac{\rho}{2}(1 - \xi^2)\right] \qquad (4.25)$$

These impose additional plastic rotations, θ_p', upon the laminae. From the geometry of the structure (Fig. 3.4.a) it follows that these are

$$\theta_p' = \frac{1}{s}\,\theta_w' \qquad (4.26)$$

The additional elastic lateral deflection at the top of the structure is

$$y_o' = \frac{\Delta W'H^3}{EI_o}\left(\frac{11}{60} + \frac{\rho}{3}\right) \qquad (4.27)$$

4.4.2 <u>The Ultimate Load on Wall 2</u>

In a similar way to the procedure followed above, W" is defined as the additional load required to enable the critical moment in Wall 2 to attain its ultimate value, i.e. $M_{2,u}$. When this load is being applied Wall 1 is assumed to accept no further load. From the compatibility of deformations over the height of the two walls, which, with the exception of the base, are still in the elastic range, it is evident that this is not possible. Wall 1, when subject to additional curvature at the upper floors, would have to resist additional load. The error involved in assuming that Wall 1 does not accept additional load, leads to a small underestimation of the ultimate load, and it affects only the equations predicting deformations. As these are a small fraction of the total deformations, the error is considered negligible with respect to the prediction of the performance of the whole structure at ultimate load. Accordingly the moments in Wall 1 do not change, and in Wall 2 they become

$$M_2(\xi) = \frac{HI_2}{I_o}[(W_y + \Delta W' + \frac{I_o}{I_2}\Delta W'')(\xi^2 - \frac{\xi^3}{3} + \rho\xi) - 1q_u\xi] \tag{4.28}$$

The maximum moment at the base of Wall 2 is

$$M_2(1) \geqq \frac{HI_2}{I_o}[(W_y + \Delta W' + \frac{I_o}{I_2}\Delta W'')(\frac{2}{3} + \rho) - .95\,1q_u] = M_{2,u} \tag{4.29}$$

The additional elastic rotations in Wall 2 are given by

$$\theta''_w = \frac{H^2 \Delta W''}{EI_2}[\frac{1}{4} + \frac{\xi^4}{12} - \frac{\xi^3}{3} + \frac{\rho}{2}(1 - \xi^2)] \tag{4.30}$$

The resulting plastic rotations in the laminae are similarly obtained from

$$\theta''_p = \frac{1}{s}\theta''_w \tag{4.31}$$

The additional deflection at the top of the structure may be conservatively estimated by

$$y''_o = \frac{\Delta W'' H^3}{EI_2}(\frac{11}{60} + \frac{\rho}{3}) \tag{4.32}$$

4.4.3 <u>The Total Ultimate Load on the Structure</u>

By superimposing the previously derived load increments the total ultimate load on the coupled shear wall structures is obtained thus

$$W_u = W_y + \Delta W' + \Delta W'' \tag{4.33.a}$$

In terms of the strength of the components of the structure this load is

$$W_u \geqq \frac{1}{H(\frac{2}{3} + \rho)}(M_{1,u} + M_{2,u} + .95\,1Hq_u) \tag{4.33.b}$$

It is to be noted that any further rotation, θ_h, which may occur in the plastic hinges, that have formed at the base of the walls, will induce proportional rotations in each of the laminae. This is $\theta_{ph} = 1\theta_h/s$ (4.34)

With this information the ductility requirement for any laminae can be found for any desired overall ductility of the coupled shear wall structure.

4.5 Illustrative Example

To illustrate the load-deformation characteristics of a coupled shear wall structure, the previously presented 18 storey shear core building (3.8) is examined a little further. Additional load increments will be applied to this structure. Each of these will correspond with a distinct stage of the behaviour. The properties required for the elastic analysis were summarised in Table 3.I of Chapter 3. Additional information related to the ultimate strength of the coupling beams is given as follows:

i) The ultimate moment capacity of a coupling beam is $M_u = 3350$ K.in.

ii) The ultimate shear strength is thus $V_u = 2M_u/s = 176K$.

iii) Correspondingly, the ultimate laminar shear is $q_u = 2 \times 176/105 = 3.35$ K/in.

iv) The equivalent moment of inertia of a cracked coupling beam is $I_x' = .3 \times 15200 = 4560$ in.^{4}.

The significant results of the analysis are presented in Fig. 4.1. The curves refer to five distinct stages of the loading till the attainment of the ultimate load on the whole structure.

4.5.1 Stage 1. Elastic Design Load

The final stage (Case C) of the elastic analysis presented in 3.8 is reproduced in Fig. 4.1 by the curves and locations labelled 1. (Also see Fig. 3.5). The maximum elastic laminar shear, $q_{max} = 2.55$ K/in., occurs at about the 4th floor (i.e. $\xi = .70$).

4.5.2 Stage 2. The Elastic Limit of the Structure

The maximum load to be carried by the fully elastic structure is to be determined. The elastic limit is attained (4.2) when the critical lamina reaches its ultimate capacity of $q_u = 3.35$ K/in. Hence from Eq. (4.4) the maximum elastic load is found to be

$$W_e = \frac{3.35}{2.55} \times 500 = 657 \text{ K} \quad \text{and} \quad P_e = 85\text{K}$$

The corresponding maximum laminar rotation is, according to Eq. (3.14) and Eq. (4.11)

$$\theta_y = \frac{hs^2 q_u}{12EI'_x} = \frac{105 \times 38^2 \times 3.35}{12 \times 5000 \times 4560} = 1.85 \times 10^{-3} \text{rad.}$$

All rotations have been expressed in terms of this yield rotation, θ_y, in Fig. 4.1.b.

4.5.3 Stage 3. The Full Plastification of the Laminae

The load which will cause at least the upper 90% of the laminae to yield need be determined. To do this, the governing criterion need be known for the location of the lamina in which yield is just attained. (See 4.3.2)

First the critical stiffness ratio is found from Eq. (4.19) and Eq. (4.18) thus

$$\rho = \frac{P}{W} = \frac{65}{500} = .13, \quad \nu = \frac{0.25 + 0.50 \times 0.13}{0.0616 + 0.095 \times 0.13} = 4.26$$

$$\text{and} \quad Z_{critical} = \frac{4.26 - 1}{1 - 0.19 \times 4.26} = 17.2$$

This value must be compared with the actual stiffness ratio of the structure, as given by Eq. (4.16)

$$Z = \frac{3.35 \times 1890^2}{2 \times 38 \times 5000 \times 1.85 \times 10^{-3}} \left(\frac{1}{3900} + \frac{1}{5650} + \frac{168.7^2}{12.3 \times 10^6}\right) =$$

$$= 46.8 > Z_{critical}$$

Thus the uppermost laminae must just attain the yield rotation. According to Eq. (4.15.a) the responsible load is:

$$W'_y = \frac{4 \times 38 \times 5000 \times 12.3 \times 10^6 (46.8 + 1)}{168.7 \times 1890^2 \; (1 + 2 \times 0.13)} \; 1.85 \times 10^{-3} = 1086 \text{ K}$$

and $P'_y = 13 \times 1086 = 141 \text{ K}$

The plastic laminar rotations, in terms of the yield rotation, are obtained from Eq. (4.17.a) i.e.

$$\frac{\theta_p}{\theta_y} = 12.7 \; \xi^2 (\xi^2 - 3.98\xi + 2.90)$$

Curve 3 of Fig. 4.1.b indicates that $\theta_p = 5\,\theta_y$ at the 7th floor and that the laminar yield rotation extends approximately over 96% of the height of the structure.

The other, more significant, quantities at this stage of the loading are as follows:

a.) The maximum lateral deflection at the top of the structure is from Eq. (4.21).

$$y_{opy} = \frac{1890^3}{5000 \times 12.3 \times 10^6} \left[1086 \left(\frac{11}{60} + \frac{0.13}{3} \right) - \frac{168.7 \times 3.35}{3} \right] = 6.45 \text{ in.}$$

b.) The bending moment along Wall 1 is from Eq. (4.7.a)

$$M_1 = 5.52 \times 10^5 \xi (\xi - .333\, \xi^2 - .390)$$

with a maximum value at the base of

$$M_1(1) = 152200 \text{ K.in.}$$

$$M_2 = \frac{I_2}{I_1} M_1 \text{ is plotted in Fig. 4.1.a.}$$

c.) The maximum axial force in the coupled walls is approximately

$$T_{max} = .975 \times Hq_u = 6150 \text{ K}$$

In order to demonstrate the influence of the stiffness ratio, Z, defined by Eq. (4.16), stiffer walls for this example will also be briefly considered. It will be assumed that instead of 18 storeys the structure consists of only 10 storeys. Thus the total height is $H = 10 \times 105 = 1050$ in. Therefore from Eq. (4.16)

$$Z = 14.5 < Z_{critical} = 17.2.$$

Therefore Eq. (4.15) and Eq. (4.17) are now applicable and the load on the structure becomes

$$W_y = \frac{38 \times 5000 \times 12.3 \times 10^6 (0.19 \times 14.5 + 1)}{168.7 \times 1050^2 (0.0616 + 0.095 \times 0.13)} \times 1.85 \times 10^{-3} = 1178\,K$$

and $\quad P_y = 0.13 \times 1178 = 153\,K$

The plastic laminar rotations are given by

$$\frac{\theta_p}{\theta_y} = 4.2\,\xi^4 - 16.9\,\xi^3 + 11.2\,\xi^2 + 0.5$$

and these are shown by the dotted curve in Fig. 4.1.b. It is evident that the plastic rotations extend over 95% of the height. They are larger at the top and smaller at midheight of the structure when compared with the rotations of the 18 storey building.

4.5.4 Stage 4. The Ultimate Strength of Wall 1 is Attained

Wall 1 is reinforced in such a way that it can develop an ultimate moment of $M_{1,u} = 200000$ K.in. in the presence of 6150 K. tension. The additional overturning moment on the structure has to be resisted by cantilever actions of the walls only because the laminar strength was assumed to be exhausted at the end of load stage 3. Using Eq. (4.22) the additional triangular load, $\Delta W'$, to be carried by the structure, when the ultimate strength of Wall 1 is attained, is found to be 118 K. Similarly $\Delta P' = 0.13 \times 118 = 15\,K$.

These loads impose additional plastic rotations upon the laminae, which, in terms of the yield rotations are, according to Eq. (4.25) and Eq. (4.26)

$$\frac{\theta_p'}{\theta_y} = 1.37\xi^4 - 5.50\xi^3 - 1.07\xi^2 + 5.20$$

When these are added to the rotations obtained in Stage 3, curve 4 of Fig. 4.1.b results. The diagram shows that a relatively small load increment (133K) resulted in very large plastic laminar rotations, particularly at the upper storeys of the structure.

The increase of deflection at the top of the structure is from Eq. (4.27)

$$y_o' = \frac{118 \times 1890^3}{5 \times 10^3 \times 12.3 \times 10^6}\left(\frac{11}{60} + \frac{.13}{3}\right) = 2.93 \text{ in.}$$

As both walls are still assumed to behave elastically, the maximum moment at the base of Wall 2 is by proportion

$$M_2 = \frac{I_2}{I_1}M_1 = 545000 \text{ K.in.}$$

The moments over the remainder of Wall 2 are plotted in Fig. 4.1.a.

4.5.5 Stage 5. The Ultimate Strength of Wall 2 is Attained.

The reinforcement in Wall 2 is such that in the presence of 6150K. axial compression an ultimate bending moment of 650000 K.in. can be developed. Therefore the additional load, which is solely to be carried by Wall 2, is from Eq. (4.28) or otherwise

$$\Delta W'' = 70\text{K} \quad \text{and} \quad \Delta P'' = 9^{\text{K}}$$

The total ultimate load carried by the structure is thus

$$W_u = 1086 + 118 + 70 = 1274\text{K (triangular load)}$$
$$P_u = 141 + 15 + 9 = 165\text{K (point load)}$$

These correspond with a load factor of 2.54.

From Eq. (4.30) and Eq. (4.31) the additional plastic laminar rotations are estimated as follows:

$$\frac{\theta_p''}{\theta_y} = 1.12\xi^4 - 4.46\xi^3 - 0.87\xi^2 + 4.22 = 0.813\,\frac{\theta_p'}{\theta_y}$$

The total laminar rotations are shown by curve 5 in Fig. 4.1.b. The graph reveals that in order to develop the full strength in this coupled shear wall structure the coupling beams at the midheight of the building need to have a ductility factor of at least 12.

The additional deflection at the top of the walls is, from Eq. (4.32) $y_o'' = 2.38$ in.

The load-deflection relationship with respect to the top of the structure is shown in terms of the ultimate load, in Fig. 4.1.c. When this continuous relationship is replaced by a bilinear one (shown dotted) it is seen that at the attainment of the ultimate load a ductility factor of 2 would give an acceptable measure of the overall plastic behaviour. A doubling of the lateral displacement of the structure, corresponding with a ductility factor of 4, would involve rigid body rotations of the walls, i.e. numerically

$$\theta_{wr} = \frac{\Delta}{H} = \frac{11.7}{1890} = 6.20 \times 10^{-3} \text{ rad.}$$

which would generate additional plastic laminar rotations. By similarity to Eq. (4.31) these can be determined as follows

$$\frac{\theta_{pr}}{\theta_y} = \frac{1}{s}\frac{\theta_{wr}}{\theta_y} = \frac{168.7 \times 6.20 \times 10^{-3}}{38 \times 1.85 \times 10^{-3}} = 14.9$$

The example demonstrated that for the particular shear wall structure the requirement of an overall ductility factor of 4 - commonly used in earthquake resistant design - would necessitate that the critical coupling beams possess a ductility factor of over 27.

4.6 A Summary of Postelastic Behaviour

It was shown in this chapter that by means of a step by step procedure it is possible to trace the postelastic performance of a coupled shear wall structure. The analysis demonstrated that very large plastic deformations are imposed upon the coupling beams even if only moderate overall ductility is to be attained.

In connection with the ultimate strength of coupled shear wall structures, three points deserve attention.

a.) Very little is known about the interaction of moment, shear and axial tension. This load combination always occurs in one of the coupled walls. The performance of such a wall may be considerably affected by the gravity load intensity, which in turn may influence the sequence of plastic hinge formation.*

b.) In the subsequent chapters a considerable amount of experimental evidence is offered with respect to the behaviour of relatively deep coupling beams. The means by which the ductility of such beams may be increased have not been explored as yet. It is questionable whether plastic deformations of the magnitude indicated in the example can be developed.

c.) The ultimate load derived analytically need be compared with the strength of the foundations. It may well be possible that the ultimate strength of the structure corresponds to forces which would violate the requirements of overall stability.

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* Research work is continuing at the University of Canterbury to explore theoretically and experimentally this load combination of reinforced concrete beams.

CHAPTER FIVE

TEST SPECIMENS, MATERIALS, LOADING FRAME AND TESTING PROCEDURE

5.1 The Test Specimens

To be able to reproduce the loading and boundary conditions which are likely to occur in coupling beams of real shear wall structures, test specimens of the form shown in Fig. 5.1 were chosen. The thickness of the central portion of the specimen, which was the test beam proper, was 6 in. The beam proper connected two 8 in. thick square shaped end-blocks. These simulated portions of the coupled shear walls. The load was applied through these end-blocks.

The load points on the end-blocks were considered to be located far enough from the beam proper so as not to cause a stress pattern at its boundaries, significantly different from that occurring in a real shear wall structure.

The dimensions of the specimens were made large enough so that no scale effects[57,58,67] had to be considered in the interpretation of the test results. The relatively large specimens also enabled numerous strain measurements to be made with relative ease on both sides of the beam proper.

All beams were cast from one side. Therefore the bond conditions for both the top and bottom reinforcement were the same.

The depth to width ratio of the deeper beams was larger than what would commonly occur in real structures. With one exception this did not seem to have had an effect upon the performance of the test beams. In Beam 394 an attempt was made to confine the concrete by means of special reinforcement.

In a wider beam this confinement would have been more effective. At no stage of the test were signs of lateral instability observed.

The 8 in. thick end-blocks were reinforced in such a manner that the estimated steel stresses were of the order of 20000 p.s.i. when the ultimate strength of the beam proper was attained. In the vicinity of the load points, where bearing pressures of up to 3200 p.s.i. intensity were produced, cross ties were provided to prevent possible splitting of the concrete.

The reinforcement in the end-blocks was not subject to test examination. The general arrangement of this steel is shown for typical beams in Fig. 6.1 and Fig. 7.43.

The end-blocks performed satisfactorily during the tests. Only at high load intensities did a few narrow diagonal cracks appear. After load removal they closed and could not be detected by the naked eye.

With the chosen loading arrangement equal moments at both ends of the beam proper and a known constant shearing force could be applied. Details of the moment pattern and the type of loading are recorded in Fig. 5.1, where the effect of reversed loading is indicated by the dotted lines.

5.2 The Concrete

5.2.1 Ingredients and Mix Proportions

Because of the limited laboratory facilities to produce larger quantities of concrete, the same was obtained from a local "ready mix" plant. The concrete used in the test series represents a typical mix which is being used for general building construction in the Christchurch, New Zealand, area.

Owing to variations in the supply of aggregates and cement over a period of 14 months, the inevitable delay in the testing of individual specimens and the variable curing conditions in the laboratory, there was considerable scatter in the values of concrete strength at the time of testing. Apart from one preliminary test (Beam 241) a 28 days cylinder crushing strength of 5000 p.s.i. was aimed at. With the excellent quality of aggregates available in the Canterbury district, it is in fact difficult to obtain a concrete strength less than 5000 p.s.i., when not less than 500 lbs. of cement per cu.yard of concrete is used and when the workability corresponds with approximately 3 in. of slump. When specifying a crushing cylinder strength of 5000 p.s.i. the following aspects were considered:

a.) The specified minimum 28 days strength for concrete currently used in New Zealand for multistorey buildings is 3000 to 3500 p.s.i. It is reasonable to expect that the actual mean strength at the age of one year or later is in the vicinity of 5000 p.s.i.

b.) To speed the early removal of formwork it is becoming a common practice to use a concrete with a higher strength than that required by strength computations.

The unit weight of the concrete was 147 ± 2 lbs. per. cu.ft. No additives were in the mix. "Guardian" type ordinary portland cement was used for all specimens.

The proportions of ingredients and other relevant data for 9 mixes used during the tests are assembled in Table 5.I.

5.2.2 Placing, Compaction and Stripping of the Concrete

The concrete was placed into the formwork immediately after its delivery. The first and last barrows of fresh concrete were not used in the beam proper but were placed in the end-blocks.

A common immersion type vibrator was used for compaction. The placing of the fresh concrete was completed within one hour. When the excess water from the surface evaporated a smooth finish was given with a steel trowel.

Together with the test specimen six standard 12" x 6" cylinders, six 6" x 6" x 6" cubes, six 12" x 3" x 3" blocks for modulus of rupture tests and two or three 18" x 6" x 6" prisms for temperature control blocks and Young's Modulus test were also cast. These were compacted on a vibrating table.

All concrete was cured for seven days under soaked sacks and polythene sheeting. The small blocks were taken out of the steel moulds approximately 48 hours after casting. The beam specimen was lifted from the formwork on the 8th day and was kept in a vertical position in the laboratory until it was placed in the test rig. The small test pieces were always stored adjacent to the beams to ensure similar curing conditions.

5.2.3 Concrete Strength Properties

The standard test specimens were crushed during or at the end of the main test. The latter lasted up to four days. The strength properties, given in Table 5.II, were generally derived from six specimens. For the 31 and 39 in. deep beams only four cylinders were used whenever a Young's Modulus Test was carried out, as this required two cylinders. For the 24 in. deep beams 18" x 6" x 6" prisms were used to determine the value of E_c. The strains were measured by Demec mechanical strain gauges over four inch lengths at midheight on four sides of the prisms or standard cylinders. Typical stress-strain curves, corresponding with short term loading of the concrete used, are shown in Fig. 5.2.

A good agreement was obtained between the measured values of Young's Modulus and the value predicted by the equation

$$E_c = 33w^{1.5}\sqrt{f'_c}\ \text{psi.}$$

of the current A.C.I. Building Code, using w = 147 for the unit weight of the concrete. (See Table 5.II).

5.3 The Reinforcement

5.3.1 Steel Properties

All reinforcement, except the # 2 bars used for stirrups in Beam 241, consisted of deformed bars. These conformed with the requirements of ASTM A 305. The #3 reinforcement was manufactured in Japan. All other deformed bars were supplied by the Pacific Steel Co. of New Zealand. This reinforcement is marketed with a guaranteed minimum yield strength of 33 Ksi., the actual yield strength being approximately 45 Ksi. The Japanese #3 bars had a yield strength of 56 Ksi.

The strength properties in each batch of delivered reinforcement were determined on 3 to 6 specimens. The large diameter bars were machined to approximately $\frac{1}{4}$ or $\frac{1}{2}$ in. diameter, to enable a Baty type extensiometer, capable of receiving bars of up to $\frac{5}{8}$ in. diameter, to be used. On companion undisturbed samples, the yield point was determined from the plot of the testing machine. The effective cross sectional area of each deformed bar was based on approximately 12 measurements taken along its 15 to 18 in. length.

Typical stress-strain curves for the bars used in the test series are assembled in Fig. 5.3. With slow application of the load at yield the beginning of the strain hardening could also be determined.

The value of Young's Modulus varied slightly for bars taken from the same batch. The mean value was 29,500 Ksi and this was used in all subsequent computations.

The #5 stirrup reinforcement was heated cherry red so as to enable neat bends to be made around the #7 and #8 main bars. A number of test specimens were prepared and subjected to the same heat treatment. It was found that the limit of proportionality was lowered to 33-37 Ksi, the yield strength to 38-40 Ksi and the ultimate strength to 62 Ksi.

It is not likely that the heat treatment, applied only locally, affected the performance of the stirrups in any way. Yielding of the stirrups was generally observed to occur away from their corners.

The strength properties of the reinforcement and other relevant data are collected in Table 5.III.

5.3.2 The Assembly of Reinforcement

The reinforcing cages for the beam proper were carefully assembled so that the main bars and stirrups were always within $^1/16$ in. of their specified positions. In order to maintain the relative positions of the bars during concreting, the stirrups were tack welded to the outer layers of main bars. The intermediate or secondary horizontal bars were tack welded to the inside of those stirrups along which no strain measurements were made.

After the completion of the cage for a beam proper and the cages for the two end-blocks, they were correctly positioned in the formwork and the anchored ends of the main reinforcement were tack welded, through cross ties, to the reinforcement of the end-blocks.

Before the assembly started ¼ in. diameter steel studs, holding future gauge points, were welded to one longitudinal rib of all those deformed bars along which strain measurements were to be made.

A plastic tube was placed over each stud and the base of this was sealed with a bituminous compound against the entry of fluids. Over the plastic tube an 11/16 in. diameter steel tube was placed and sealed in a similar manner. The small spaces at the inside and at the outside of the plastic tube were filled with hot wax after the reinforcing cage was fully assembled.

A few days after the test specimen was lifted from its formwork, the wax was broken and the plastic tube was removed so that an approximately 1/8 in. gap was left between the inside of the steel tube and the central steel stud. Fig. 5.4 shows a close up of the stud-tube assembly before concreting. On Fig. 5.5 the position of the studs within the steel tubes may be seen.

This arrangement ensured that the surrounding concrete did not interfere with the steel strain measurement even when the strain hardening range was entered. The lengths of the studs varied according to the position of the instrumented reinforcement so that the ends of the studs extended to within 1/32 in. of the finished concrete surface.

5.4 The Loading Frame

The welded steel loading frame, with all its attachments, was made in the workshop of the Department of Civil Engineering. When the frame was designed the following requirements received consideration:

a.) A self-equilibrated and statically determinate loading system was desirable as this avoided having to transmit large forces to the floor of the laboratory.

b.) Only two 100 Ton capacity hydraulic jacks were likely to be available.

c.) It should be possible to reverse the direction of load speedily.

d.) The sides of the beam proper should be free from obstructions to enable measurements to be made comfortably from both sides.

e.) Specimens of different heights should be easy to accommodate.

f.) Assembly and disassembly should not be time consuming.

A diagrammatic representation of the essentials of the frame assembly is shown in Fig. 5.6. Details may be seen on the photographs reproduced in Fig. 6.3 and Fig. 6.81.

With reference to Fig. 5.6, the working of the testing frame may be briefly summarised as follows:

(A) The load was applied by means of a 100 Tons capacity hydraulic jack, which was placed between the two halves of the welded steel frame. The line of action of the applied force passed through the centre line of the beam proper.

(B) For reversed loading the jack was placed upon the top of one half of the frame and by means of cross-heads it was tied, with high strength steel bars, to the under-side of the other half of the loading frame. When two jacks were available only the inner jack had to be removed at certain load cycles. (See Fig. 6.81).

(C) The frame was attached to the test specimen by pairs of rolled steel channel stirrups. A rigid steel bearing show was welded between each pair of channels at the underside of the test specimen. At the top of the steel frame $^5/8$ in. thick slotted bearing plates were connected to the channels. Here $1\frac{1}{8}$ in. diameter High Strength Friction Grip Bolts were used. Numerous holes through these channels enabled different sizes of test pieces to be accommodated under the frame.

(D) The load to the test beams was transmitted either through the rigid shoes between the stirrups or through the stiffened bearing plates at the underside of the loading frame. With the exception of the shallow beams (241 to 244), the approx. $\frac{1}{8}$ in. wide gap between concrete and steel bearing plates was filled with plaster of Paris. For the shallow beams $1\frac{3}{4}$ in. thick three layered Neoprene bearing pads were used. These were found to cause excessive rotations of each half of the testing frame. Consequently the line of action of the load moved, especially at high load intensities, away from the desired centre line. In the later tests the plaster fill enabled the eccentricity to be maintained within $\pm\frac{1}{8}$ in.

(E) The whole assembly was supported by small cross-channels which were attached to the inner (double) stirrups. They transmitted the whole weight of the assembly to the floor. The connections at two feet from both sides of the vertical plane of the assembly provided lateral stability during testing.

(F) Four levelling bolts at each support point enabled the assembly to be erected in a truly vertical position. During testing these bolts did not engage.

(G) To enable rotations during the testing to occur freely the left hand support of the assembly was seated upon an adjustable knife edge.

(H) A similar roller support at the right hand ensured that no horizontal restraint existed. Up to $\frac{3}{8}$ in. lateral movement of the roller occurred during tests.

(J) Lubrucated tongues, sliding in corresponding slots, ensured that the two halves of the loading frame remained in the same vertical plane during the test.

5.5 Instrumentation

5.5.1 Steel Strain Measurements

Demec mechanical strain gauges were used to measure the elongation of the reinforcing bars over a 4 inch base length. Drilled stainless steel plates, which received the points of the strain gauges, were attached to ¼ in. diameter steel studs. The latter were welded to the reinforcement as described in 5.3.2. Prior to testing the small holes in the stainless steel plates were thoroughly cleaned with a drill and air jet and each was examined under a microscope.

5.5.2 Concrete Strain Measurements

Generally 4 in., and occasionally 2 in. base lengths were used for Demec strain gauges on both sides of the beams. The concrete surface was cleaned with a wire brush and the dust was removed with an air jet. The gauge points were attached to the concrete surface with sealing wax either before the test or after the formation of the cracks.

5.5.3 Rotation Measurements

The horizontal displacements along three points of a vertical reference line, passing through the centre of the end blocks, were measured by dial gauges. Small steel plates, attached to the concrete surface through a short arm, received the points of the dial indicators. The general arrangement may be seen in Fig. 6.3 and Fig. 6.81.

The readings near the top and bottom edge of the end blocks were used to determine the absolute rotations. The centre reading provided a check to see if the three chosen points remained indeed on a straight line during the test. Only at high load intensities, when one or two diagonal cracks crossed the vertical reference line, could a small deviation of the centre points from the line connecting the other two points be observed. The effect of this deviation was neglected.

At high load intensities the rapid creep rotations overshadowed the above deviation.

Readings taken at the centre points of the end blocks, which were situated on the horizontal axis of the beam proper, enabled the displacement of one end-block relative to the other to be determined.

From the measurements on both sides of the test specimen any twisting, which may have occurred, could be detected. For the purpose of computation the average of the measurements, taken on both sides, was considered.

To enable the load-rotation relationship for the beam proper to be established, it was necessary to determine the "rigid body rotation" of the whole test piece. This rotation resulted from:

a.) The rotation of the end blocks about their respective supports. These were not situated under the centre reference line of the blocks. (See Fig. 6.81)

b.) The elongation of the channel stirrups of the loading frame. These were stressed to 22 Ksi at high load intensities.

c.) The deformations of the loading frame.

d.) The compression of the plaster of Paris or Neoprene packing between the steel bearing plates and the concrete surface.

e.) The vertical strains in the concrete of the end blocks.

This "rigid body rotation" was appreciable, and it was necessary to determine it accurately as it affected all other dial gauge readings of the test. For this reason a dial indicator was placed under the bottom surface, at the centre line of the end blocks, one inch away from each of the two edges. An additional gauge was placed half way between the previous two dial indicators. So under the centre line of each end block the deflection was measured on three dial gauges placed 3 in. apart from each other. The average of the outer readings was compared with the value shown on the central dial gauges and in this way any erroneous measurement could be quickly detected and corrected.

This seemingly overcautious arrangement was necessitated by the disappointing results obtained in the first tests, where this vital information was lost at some load increments. The two outer dial gauges also enabled the undesirable tilting of the end blocks to be observed.

The accuracy of the readings on these dial gauges, was approx. .0003 in.

5.5.4 <u>Measurements of Beam Deflections</u>

In order to get the general picture of the deformations of the beam proper, usually 9 dial gauges were placed under the centre of its soffit. This may be seen in Fig. 7.37. The accuracy of the readings on these ½ in. travel dial gauges was approx. 0.00003 in. In the interpretation of the results due allowance was made for the rigid body rotations of the specimens.

5.5.5 <u>Load Application</u>

The nominal 100 Tons capacity hydraulic jacks were fed by a 10,000 p.s.i. capacity Riehle testing machine, which maintained constant oil pressure over the 21.2 sq.in. of ram area. This arrangement of the loading was compared with another testing machine. The accuracy of the load was within 1%, except when the oil pressure was less than 500 p.s.i. i.e. at loads less than 10 Kips. The duration of the constant loading varied between 2 and 40 minutes.

By means of a stretched string the line of action of the load was frequently compared with the line passing through the theoretical point of contraflexure of the beam proper. Any deviation, which may have occurred, was recorded.

The difficulty in following strains and displacements near the failure load was a disadvantage of this "constant load" device. Often the test specimen "ran away" and it was destroyed before the yield displacements could be arrested by reducing the load slightly. In spite of applying very small load increments, when approaching the estimated failure load, only limited information could be obtained with respect to the ductility of coupling beams. Unfortunately a 100 Tons capacity load cell, which would have enabled a "constant displacement" form of load application to be used, was not yet available at the time of the experiments.

5.5.6 Crack Obsverations

The cracks were located and their development was followed after each significant load increment with the aid of magnifying glasses. The propagation of cracks was marked. When a significant change occurred in the crack pattern a photograph was also taken from each side of the beam. A few of these are reproduced in this report.

On the surface of a few beams the width of some typical cracks was followed with a microscope.

These coupling beams were considered as parts of an "earthquake resistant" structure. Their performance, including crack development, could not be related to a service load. For this reason in most beams no attempt was made to accurately observe the widths of the cracks.

5.5.7 Temperature Control

As constant temperature could not be maintained in this laboratory, it was considered advisable to assess the effects of a variable temperature upon the strains in the test piece. For this reason one 6" x 6" x 18" concrete control block was provided with two 4 in. and two 2 in. gauge lengths.

Measurements on these temperature control blocks confirmed previous experience in the laboratory, that generally these blocks responded faster to temperature variations than large test specimens. Therefore an undisturbed area of a previously tested beam was also instrumented for temperature control.

Fig. 5.7 shows the typical evaluation of temperature effects for a beam (312). This specimen was subject to alternating loading over four days. The mean of the readings taken on the control block and on the companion beam was replaced by a simple curve or straight lines. The actual correction to be applied to the steel or concrete strain readings was that which corresponded with the mean time of the particular load level. This is shown by the vertical lines and arrows in Fig. 5.7 for the 3rd and 4th day of the test.

It is to be noted that the scatter of strain readings for both concrete and steel, at high load intensities, was generally much greater than the possible strain variation due to temperature changes. Thus the temperature corrections were considered to have affected significantly the strain readings at very low load intensities only, i.e. when the beams were still uncracked.

5.6 The Testing Procedure

At the commencement of each test 2 to 3 and occasionally more readings were made at each strain or dial gauge so as to establish the mean value of the "no load" condition. After the load was applied to the test specimen the measurements were generally taken in the following sequence:

1.) Checking the line of action of the load.

2.) Temperature and time.

3.) Demec gauge readings on the standard bar and temperature control blocks.

4.) Oil pressure on the hydraulic jack.

5.) Dial indicators for rotational measurements.

6.) Strains along the flexural reinforcement and intermediate horizontal bars.

7.) Stirrup strains.

8.) Concrete strains.

9.) Dial indicators at the soffit of the beams.

10.) Repeated readings for rotational measurements as in 5.

11.) Locating and marking of cracks on both sides of the beam. Crack width measurements when necessary.

12.) Reduction of the load when its intensity was high.

13.) Photographs taken when necessary.

For each pair of gauge points the readings were taken simultaneously on both sides of the specimen. All readings were manually recorded on standard data sheets and from these the strains, with all necessary corrections, were processed by an electronic computer.

CHAPTER SIX

MEDIUM COUPLING BEAMS

This test series consisted of four beams, designated 311 to 314. Their properties are given in Chapter 5. The overall dimensions of the beamswere the same. The span to depth ratio was

$$\boxed{\frac{l}{D} = 1.29}$$

The web reinforcement and the load sequence have been varied.

6.1 Beam 311

6.1.1 Loading and Test Procedure

The beam was loaded in several increments till destruction. The aim of this test was to determine the formation of a failure mechanism in a coupling beam, deliberately underreinforced against shear. Numerous measurements of strains and displacements were made during the test. Certain features and also difficulties encountered are discussed in detail. Some of these features also occurred in other test specimens. Therefore reference will be made to them in the description of the behaviour of other beams.

The load sequence and other data related to loading is presented in Table 6.I.

The overall view of the reinforcement for the whole test specimen is shown in Fig. 6.1 and a detail of the beam proper with the welded studs for strain measurements in their sealed protective tubing may be seen in Fig. 6.2.

The beam, set up in the test frame for one way loading, is shown in Fig. 6.3. The photograph was taken after the completion of the test when some of the dial gauges used were already removed. Fig. 6.4 shows the crack pattern of the beam after failure. The gauge points along the flexural and web reinforcement and some of the gauge points on the concrete can also be seen on this photograph. As a rule all photographs presented here view the beams from the East (note the letter E) and references, such as "left hand support" apply to this side of the beams.

6.1.2 The Behaviour of the Flexural Reinforcement

6.1.2.1 The distribution of strains along the flexural reinforcement.

The generous provision of gauge points enabled the strains to be assessed over the entire clear span of the beam proper. These strains are plotted in Fig. 6.5 for various load levels, which have been expressed as a fraction of the ultimate load. At each load the strain distribution is shown separately for the outer layer (#8) and for the inner layer (#7) of bars. The points on the graph represent the average strain in the two bars of the same layer.

Because of the random formation of cracks frequently considerable differences were observed between the strains measured at one and the other side of the beam at the same section. A crack extending across the 6 inch width of the beam did not necessarily cross corresponding gauge lengths. A point omitted from a graph indicates that the gauge point hasmalfunctioned or has fallen off.

From an examination of the strain distribution curves for the top and bottom reinforcement, Fig. 6.5, the following observations may be made.

a.) The strains in the two layers of reinforcement at the top or at the bottom of the beam are generally very similar. They do not appear to have a set relation to each other. Mostly the strains in the outer bars (# 8) are somewhat larger, as one would expect from a linear distribution of strains across a section.

b.) Up to about 35% of the ultimate load the strain distribution corresponds with the bending moment pattern. (See Fig. 5.1). The strains are very small at the centre line of the beam, i.e. at the point of zero moment. As expected the compression steel strains are considerably smaller than the corresponding tensile strains.

c.) At higher loads, which are associated with the development of diagonal cracks, the tensile strains become rather large also at the point of zero moment. Already at 58% of the ultimate load tensile strains were observed over the entire length of the beam in the top and in the bottom bars. Surprisingly, at higher loads the flexural reinforcement was found to be in tension also in the "compression zone" of the beam.

d.) Near ultimate load the tensile strains in the two layers of reinforcement passing through the "compression zone" of the beam differed greatly. This unexpected phenomenon is associated with the failure mechanism of the beam. It is discussed more fully in 6.1.5.

e.) The strain distribution along the top and bottom reinforcement was, at all load levels, reasonably antisymmetrical. The deviations which occurred are due to the irregular crack formation rather than due to a change in the bending moment pattern. The latter may have resulted from a small change in the alignment of the load line.

The line of action of the applied force was frequently checked against the vertical centre line of the beam. Occasionally, at high loads when considerable rotations of the end blocks occurred, a small deviation could be observed. An eccentricity of $^3/16$ in., a typical quantity, represents approximately 2% difference between the static moments generated at the supports of the beam. During the elastic performance of the beam such discrepancies were of no significance. However at high loads they could have been responsible for unsymmetrical plastic deformations at the supports. Such eccentricies, when observed, were recorded and in subsequent tests they were largely eliminated at zero loads by adjusting the position of the jack.

6.1.2.2 The tension force distribution.

The behaviour of the beam and the mode of load resistance can be better visualised if the previously recorded elastic strains are translated into internal forces. By taking the mean strain for the four bars in each face of the beam and by using 29,500 Ksi for the modulus of elasticity of the steel, the tension force distribution curves for five load increments have been obtained as shown in Fig. 6.6.

The deviation of this tension force distribution along the beam from the pattern that would result from a conventional analysis can be better appreciated if the latter is also determined. By considering a singly reinforced concrete section and taking the modular ratio $n = 7$, the internal lever arm is found to be 24.7 inches. The lever arm of the steel couple is 25.2 in. For the purpose of this comparison an internal lever arm of $z = 25$ in. was used in all beams of the series.

The theoretical steel force distributions, which result from this analysis in the tension zone of the beam are shown by the dotted lines in Fig. 6.6. The discrepancy between the theoretical and measured tensile forces, particularly at higher loads, is apparent. The causes for this are examined at the end of this chapter. (6.5.1.)

6.1.2.3 The position of the internal forces.

The performance of the flexural reinforcement was fully assessed in the experiment. Therefore it was possible to locate the internal stress resultants. A study of the position of the internal forces highlights the difference between the behaviour of a normal, slender beam, and that of a deep coupling beam.

With reference to Fig. 6.7 the following quantities may be identified:

T_t and T_b are the tensile forces induced in the top and bottom reinforcement respectively as given in Fig. 6.6.

From $T = T_t + T_b = C$ the position of the total tensile force, T, is found thus:

$$e_s = \frac{T_b}{T} d''$$

The total tensile force, T, generated along the test beam at various load levels is also presented in Fig. 6.8. Thus at about half the ultimate load uniform tension (hence compression) existed along the beam and at higher loads, this tension increased further at the centre of the beam where, according to conventional theory, it should have been zero.

The bending moment is known from the applied load, P_i, i.e. $M = P_i x$, where x is the distance measured from the centre of the beam.

The compression resultant may therefore be located from

$$z = \frac{P_i}{T} x \quad \text{and} \quad e_c = z + e_s$$

The results of the above simple computations, as applied to Beam 311, are presented in Fig. 6.9. The diagram shows the position of tensile resultant, T , and the position of the compression stress resultant, C , along the beam for a number of load levels. The following points may be noted with respect to the cracked beam:

a.) The internal lever arm, z , is not constant over the length of the beam. It reduces from a maximum at the supports to zero at midspan.

b.) The internal lever arm reduces at all sections as the load increases.

c.) The line of thrust, i.e. the line passing through the positions of the compression stress resultants, does not appreciably change with the load.

d.) It is the change in the position of the tension resultant which is responsible for the reduction of the internal lever arm. This is brought about by the gradually increasing tension in the "compression reinforcement".

e.) At a load less than approximately one half of ultimate, the cracks in the beam have not yet fully developed. In parts the concrete still carried a small amount of tension. This was ignored in the computation but it has influenced the results for the computed internal lever arm. (See the dotted line for $P_i = .46\ P_u$)

The variation of the actual internal lever arm at sections of the maximum moments is expressed in terms of the theoretical value in Fig. 6.10. There is a considerable scatter, but the general trend indicates that at ultimate load the internal lever arm is only about 65% of the value predicted by conventional elastic analysis or by ultimate flexural load theories. (z at ultimate "Whitney" load is 25.8 in.) It is to be noted that the lever arm, which has been examined above, refers to the total tensile force, T.

There is also a discrepancy between the stresses in the tensile reinforcement ($f_s = T_t/A_s$ or T_b/A_s) when conventional theory and experimental results are compared. Fig. 6.11 indicates that the more fully the diagonal cracks develop, i.e. at over 60% of the ultimate load, the more do the tensile stresses rise above the value predicted by conventional theory at the sections of maximum moments. The maximum deviation for Beam 311 was 13% at the premature onset of yielding.

6.1.3 <u>The Behaviour of Stirrups (# 3)</u>

6.1.3.1 Load-strain relationship.

In previous tests it was found that, because of the unpredictable formation of cracks, strain measurements made at midheight of stirrups, do not necessarily supply the most important information, the value of the largest strain. For that reason, in this series of tests, 70 to 85% of the height of every second stirrup was covered by strain measurements at both sides of the beams. In Beam 311 the main diagonal crack has nevertheless bypassed the instrumented length of the first and last stirrup. See Fig. 6.4.

Some results of these strain measurements are shown in Fig. 6.12. The key diagram indicates the five gauge positions (numbered 1 to 5) for the five instrumented stirrups (numbered 5 to 9). The heavy lines indicate the length of each stirrup over which the stresses are reproduced in the other four diagrams of this figure.

It is to be noted that strains at cracks, particularly after the onset of yielding, may be considerably larger than the recorded strains. The latter were obtained by measuring the elongation over four inch length of a bar.

The load-stress relationship shown in Fig. 6.12 for stirrups 5, 6, 7 and 8 clearly show that stresses vary considerably along each stirrup. After the development of diagonal cracks the most highly stressed region of each stirrup occurred where it was crossed by the main diagonal crack. This crack crossed the beam from the lower left hand to the upper right hand corner. (See Fig. 6.4). Its position is also sketched on the key diagram. It is most probable that stresses below gauge point 5 of stirrup 5 were considerably higher than those indicated by cirve 55.

All load-stress curves confirm the well established fact that stirrups commence to contribute towards shear resistance only after they have been crossed by diagonal cracks. Before the formation of cracks negligible stresses, often crompression, was detected in the stirrups. From the sudden change of direction of the curves the diagonal cracking load could be estimated at 66^K. (v_{dc} = 395 p.s.i.)

6.1.3.2. A comparison with the equation of the American Concrete Institute.

The contribution of stirrups towards shear resistance in coupling beams can be better appreciated if this is related to some existing theory or design procedure. For this reason the current A.C.I. recommendations[59] will be considered. These have been derived from a very large volume of experimental evidence. However, to the writer's knowledge, no beams similar to the ones reported here have previously been examined. It is therefore interesting to study the applicability of the A.C.I. design procedure with respect to the shear strength of coupling beams.

The ultimate nominal shear stress which the shear resisting mechanisms of a reinforced concrete beam, excluding the contribution of web reinforcement, are assumed to supply is given by Eq.(17-2) of the A.C.I. Code;

$$v_c = (1.9\sqrt{f'_c} + 2500\, p_w \frac{Vd}{M})\, \Phi$$

By substituting the appropriate values for Beam 311 and by taking Vd/M conservatively as unity, it is found that $v_c = \Phi 179$ p.s.i. When, for the sake of comparison with actual load, the capacity reduction factor, Φ, is taken as unity, the shear force resisted by the concrete beam without web reinforcement is found to be $V_c = 30.0$ Kips.

The contribution of the stirrups at yield is

$$V_s = \frac{a_v f_y d}{s} = \frac{2 \times 0.1065 \times 56.0 \times 28.1}{4} = 83.4^K$$

so that the total ultimate shear force predicted by the code is

$$V_u = V_c + V_s = 113.4^K = .77 P_u = .72 P_u^*$$

It was therefore hoped that shear rather than flexure would govern the ultimate load on the beam.

Generally the view is held that V_c conservatively assesses the diagonal cracking load, which beams of normal proportions are capable of sustaining till the ultimate strength of the web reinforcement is attained. It is interesting to observe that, though the actual cracking load was about twice as much as V_c, the ultimate capacity of the inner stirrups was attained as predicted by the code equation.

The components of the shear resisting mechanism in beams of normal proportions and without web reinforcement have been identified and studied by Fenwick.[60] It was also postulated[61] that in beams with small shear-span to depth ratio, considerable shear may be supported by arch action, but only after extensive diagonal cracking. One might be inclined to think that this is the principal mode of shear resistance in these short coupling beams. It will be shown later that, because of the nature of load application, the contribution of arch action diminishes in these beams as the load approaches its ultimate value.

It is to be noted that the A.C.I. stirrup equation is based on the classical truss analogy of Moersch, who assumed diagonal cracks at 45°. In beams of this series such a crack would cross 7 stirrups. In fact the critical crack, leading to a failure mechanism has formed at angle of approximately 38° and has thus encountered 9 stirrups. On the basis of this the shear capacity of the web reinforcement is

$$\frac{9}{7} \times 83.4 = 108.0^{K}$$

This is 74% of the failure load. It is of the same order as the shear force predicted by the A.C.I. equation.

6.1.3.3 The strain distribution along the stirrups.

Contrary to the assumptions of the conventional truss analogy, the stresses are not uniformly distributed along the length of a stirrup. By means of bond considerable forces are absorbed by the surrounding concrete so that generally the stirrup stresses are smallest at the anchorages near the flexural reinforcement. When diagonal cracks open the stirrup strains increase rapidly at these cracks. Fig. 6.13 shows the strain distribution for three typical stirrups at various levels of the loading. The position of the major crack, relative to the stirrup, is also indicated. It may be observed that when yield has occurred at a particular point (see stirrups 6 and 7) the stresses in the remainder of the stirrup do not increase appreciably with the load.

6.1.3.4 The ultimate strength of the web reinforcement.

The distribution of strains along stirrups indicates how the beam approaches the failure condition. The maximum strains for each stirrup, which occur along the main diagonal, are shown in Fig. 6.14. These indicate that the central stirrups are strained at a greater rate than those near the supports. The strains for the latter stirrups had to be extrapolated from the graphs of Fig. 6.13. Failure occurred when these end stirrups too had yielded.

The forces sustained by the instrumented stirrups at the main diagonal crack at different levels of the loading have been plotted in Fig. 6.15. By assuming that the contribution of the intermediate stirrups (shown dotted) can be approximated by linear interpolation, the total strength of the web reinforcement was computed. The area under each graph is proportional to the resisting force. This total shear force carried by all the stirrups was then plotted against the applied load in Fig. 6.16. The shaded area indicates the fraction of the load resisted by the web reinforcement. The remainder was resisted by other mechanisms. The numbers on the small horizontal lines indicate the percentage of the shear carried by the nine stirrups at a particular level of the load intensity.

6.1.4 <u>Concrete Strains</u>

6.1.4.1 Compression strains in the uncracked beam.

Horizontal concrete strains were measured at a number of points situated along sections passing through the supports, at quarter points and at midspan of the beam. Two and four inch Demec gauges were used.

The strain distribution at a section passing through the uncracked compression zone at the support of the beam is shown in Fig. 6.17.a. The expected non-linear distribution of the horizontal compression strains and the stress concentration at the re-entrant corner are clearly recognisable. In this range of the loading the stresses are approximately proportional to the concrete strains. (See the stress-strain curve for concrete used in Beam 311 in Fig. 5.2). Thus it can be seen that the maximum fibre stress is about two and a half times as much as the value predicted by the classical beam theory. The strains corresponding with the latter have also been plotted for the sake of comparison.

Much better agreement with the ordinary beam theory is obtained at the quarter span sections. This is shown in Fig. 6.17.b. It is to be noted that the strains in the uncracked beam are rather small and thus the results are more affected by inaccuracies of the strain measuring process and the estimation temperature effects. To illuminate this point the temperature corrections have also been plotted to scale in this diagram for a number of load levels.

6.1.4.2. Compression strains in the cracked beam.

Considerably more difficulty was encountered with concrete strain measurements after diagonal cracks developed and crossed a large number of gauge lengths. Many strain readings became useless.

The results for strain measurements in one half of the antisymmetrical beam are shown in Fig. 6.18 for load levels higher than 35% of ultimate. The major cracks, which pass through the particular section, are also sketched. These measurements have a qualitative value, for they indicate that:

a.) The compression strain distribution is markedly different from that predicted by the conventional theories associated with cracked reinforced concrete sections.

b.) At the support section a kink occurs in the strain pattern. This is probably caused by the "compression reinforcement" which at this section is in tension.

c.) At quarter span the effect of the reinforcement is prominent at the "compression edge". Here the concrete is in tension or has cracked. Considerable compression stresses occur at the level of the axis of the beam.

d.) At midspan (point of zero moment) near-uniform compression stresses developed after diagonal cracking had occurred. Near the failure of the beam these compression stresses exceeded 2000 p.s.i. According to the theory of homogeneous, isotropic elastic beams, this section should have been in the state of pure shear. At the top and bottom of the beam the tension in the flexural reinforcement dominated the strains, so the concrete has cracked.

For the sake of comparison the line of thrust, as obtained in Fig. 6.9 by consideration of the steel forces only, has been transposed over Fig. 6.18. This is shown by the heavy line. It indicates that compression stresses at the supports extended over a considerable depth. This must have necessitated some of the cracks, which penetrated the compression zone at low loads, to close again after the formation of diagonal cracks. There was some evidence for this.

The estimation of the compression force, from the elastic compressive strains at midspan, gave a satisfactory agreement with the total steel force, T , (Fig. 6.8) across this section. (See note in Fig. 6.18).

It is to be noted that the principal, inclined compression strains are larger than those presented in this diagram.

6.1.5 The Failure Mechanism

From the photograph of the beam, taken at failure, Fig. 6.4, and from measurements made on the web it is apparent that the beam was separated into two triangular halves along the main diagonal. When, for the purpose of study, one half of this mechanism is separated, as shown in Fig. 6.19.a., the interplay of forces may be examined, by considering the equilibrium conditions for this free body. The whole external load, P_u , must be transmitted across the main diagonal crack when failure occurs. This may be achieved by the following component forces:

a.) Each stirrup, crossed by the crack, supplies a force S at yielding. There are nine stirrups involved in this beam representing the major component of the load, i.e. $S = 108^K = .74 P_u$.

b.) Owing to the vertical displacement of the two halves of the beam dowel forces D_1 and D_2 may be generated across the flexural reinforcement. The quantitative assessment of these was not attempted in the experiments.

c.) It was found that the flexural reinforcement is subject to moderate tension near ultimate load, where it projects from the triangular half of the beam. (See Fig. 6.6). As there is no horizontal load applied to the beam, a shearing force A must be induced along the diagonal so as to balance the tensile forces, T^* .

The three components of the internal resistance are also shown in a vector diagram along Fig. 6.19.a.

Before failure the main diagonal crack was small in the vicinity of the flexural reinforcement. This was evident from stirrup strain and crack width measurements. Therefore shearing forces (A) could be transmitted across the cracks in this area by the interlocking of aggregates. In the centre of the beam the crack becomes so wide that no shearing force could be expected to be transmitted there by aggregate interlocking.

When the last stirrups, near the supports of the beam, began to yield, the aggregate interlock force, A , must have also diminished. The flexural reinforcement was subjected to yield under the action of the dowel forces. Fig. 6.20 shows a close up of one corner of this beam after failure, viewed from the West. Note the displacement of ¼" studs.

Under the combined action of the above described components the triangular cantilever deforms into a shape indicated by the dotted line in Fig. 6.19.a. A comparison of the shapes of the two halves of the beam, in Fig. 6.19.b, offers an explanation for the distribution of the strains along the stirrups, where they cross the main diagonal. (See Fig. 6.14). The shape also suggests that a serious condition of discontinuity is created for the flexural reinforcement, where this crosses the main diagonal at the corners of the beam. At these points, as detailed separately in Fig. 6.19.c., the two layers of flexural bars must follow a "hinge" rotation. As a consequence of this the top bars are subject to compression and the bottom bars to appreciable tension. These stresses are superimposed upon the stresses which already exist in the bars. This local disturbance becomes very marked near failure. The phenomenon could be clearly observed (in Fig. 6.5) at both affected corners of the beam. It was repeatedly confirmed in subsequent tests.

6.1.6 <u>Deformations</u>

6.1.6.1 Rotations.

An important piece of information, which these experiments were expected to supply, was the load-deformation characteristic of the coupling beams. From this it was hoped that an appropriate value of the stiffness could be established and used in the theoretical predictions such as presented in Chapter 9.

It was decided that the rotation of the vertical sections of the beam proper, at its boundary, should be related to the load. It is to be noted that this rotation of the boundary section is not equivalent to the rotation of the tangent to the beam's axis at the boundary. Because of the shear deformations within the body of the beam proper, the angle between the tangent to its axis and a plain section across it, both considered at the boundary, will not be ninety degrees.

The desired relationships are:

$$\Phi_L = f_L(P_i) \quad \text{and} \quad \Phi_R = f_r(P_i) \quad \text{where} \quad P_i = M_i/20''$$

The terms are defined in Fig. 6.21.a.

It was hoped to account for all deformations within the beam proper and to exclude deformations which occur in the end-blocks. Therefore in the first test specimen a vertical line adjacent to the boundary of the beam, at L and R, was instrumented. This procedure failed to give useful information. The boundary section did not remain plain and the gauge points were excessively affected by cracks, which propagated past the boundary of the beam into the end blocks. (Beam 241).

It was decided to move the vertical reference line - the horizontal displacement of which was to be measured at three points - further away from the boundary into a region which would not be affected to a great extent by deformations of the concrete during the test. A vertical plane passing through the centre of the end-blocks, at points A and B on Fig. 6.21.b, gave a convenient line. There was also access to these points. Details of the instrumentation along this line were described in Chapter 5.

To obtain the desired beam rotations, Φ_L and Φ_R, in a simple way, it was necessary to assume that the end-blocks behaved as infinitely rigid bodies. The error involved led to an overestimation of the rotational capacity of the beams. The overestimation is quite significant in the uncracked state of the specimen but it diminishes as the stiffness of the beam proper, relative to the end-block, is reduced with progressive cracking. A quantitative assessment of these assumptions and the necessary corrections are discussed at the end of this section. It must be noted that the error involved is of the same order for all beams and thus it does not affect the comparison of the different load-rotation relationships.

With the aid of Fig. 6.21.b the following simple relationships can be established:

The rigid body rotation of the test specimen, which was supported at two points from the floor of the laboratory, was defined as

$$\alpha_o = \frac{\Delta_A + \Delta_B}{96}$$

where Δ_A and Δ_B were the measured displacements at A and B .

The rotations of the end-blocks, α_A and α_B, were determined from dial gauge readings. Thus the rotations of the end-blocks with reference to the base line A-B could be obtained;

$$\Phi_A = \alpha_A - \alpha_o \quad \text{and} \quad \Phi_B = \alpha_B - \alpha_o$$

The displacements of the supports of the beam proper at L and R from the base line are given by

$$\Delta_L = 28\Phi_A \quad \text{and} \quad \Delta_R = 28\Phi_B$$

The desired rotation at L is therefore the angle between the lines AL and LR, i.e.

$$\Phi_L = \frac{\Delta_L + \Delta_R}{40} + \Phi_A = 1.7\Phi_A + .7\Phi_B$$

Similarly $\Phi_R = 1.7\Phi_B + .7\Phi_A$

Because the structure is symmetrical the end-block rotations should have been identical. However, because of uneven crack formation and distribution of material properties, there was usually a small difference between the two rotations. This has become very large when yielding at one end of the beam has set in earlier than at the other end. The phenomenon is known from the tests of symmetrical continuous steel or concrete beams.

The load-rotation relationship for Beam 311, based upon measurements and the above computations, is presented in Fig. 6.22. The load was removed at the end of each day of the testing. These stages are shown by dotted lines, which also indicate the permanent deformations. The significant stages of crack development and the commencement of yielding are also recorded. These can be related to the shape of the curve. The rotations of the two ends of Beam 311 were practically identical.

In order to estimate the effect of end-block deformations upon the above derived beam rotations, particularly in the uncracked state, the deflections of the whole test specimen were studied. This theoretical examination was based on the following assumptions:

a.) The principles of the conventional elastic beam theory apply.

b.) The applicable bending moments are those defined along the horizontal axis of the specimen. This allows for a considerable spreading of the externally applied load and for a corresponding rounding off in the peak value of the moment. (See Fig. 5.1).

c.) At the junction of the beam proper and the end-block a 4 inch long (6 in. wide) transition area exists to which the average moments of inertia of the beam and the block is allocated.

d.) The flexural deformations of the end-block, and the flexural and shear deformations of the beam proper are significant only.

This study indicated that the computation based on an infinitely rigid end-block would overestimate the rotations of the beam proper by 57% when both parts are in the uncracked state. At high load intensities, when the beam proper is cracked, the contribution of the end-block deformations is less significant. The overestimation in this latter case is 5-12%. The 1.57 times magnified theoretical beam rotations, which are comparable with the experimental values, are shown by the dotted lines in Fig. 6.22. In the light of the assumptions made (a.) to d.) above) the agreement may be considered as being satisfactory.

6.1.6.2 The elongation of the beam.

If the length of a rectangular reinforced concrete beam is measured along its axis at middepth, then it may be said that all such beams increase their length under load after cracking. The reason for this is the movement of the neutral axis away from the geometrical axis. The latter is generally situated at a level where tensile strains prevail. For slender beams the lengthening is insignificant.

The mode of the load resistance in coupling beams was shown (in 6.1.2) to be such that after diagonal cracking the top and the bottom reinforcement was in the state of tension over the entire span of the beam. It is thus evident that the beam must become longer.

The phenomenon becomes more obvious when the position of the coupling beam is examined in relation to the adjoining shear walls, as sketched in Fig. 6.23. If the coupling beam is incompressible the two shear walls would have to move apart by a distance $D\alpha$. However, to preserve continuity, the beam shortens along the diagonal, shown dotted, and extends in the horizontal direction.

As part of the rotational measurements the elongation of the specimen between points A and B (see Fig. 6.21) could also be determined. The elongation measured this way is larger than the extension of the beam proper, because it also includes the extensional deformations in one half of each end-block. It also includes possible slips at the anchorages of the flexural reinforcement. An assessment of the contribution of slip, anchorage deformation and elongation of the beam proper towards the total measured end-block displacement is discussed in 6.5.3.2.

The load-elongation ($P_i - \Delta_H$) curve is shown, together with the load-rotation relationship in Fig. 6.22. The curve changes its direction at each significant change in the behaviour of the flexural reinforcement. The curve indicates more clearly than the load-rotation graph when flexural cracking commenced.

The $P_i - \Delta_H$ relationship was found to give useful information with respect to the plastic deformations of the flexural reinforcement.

6.1.6.3 Transverse expansion.

The deformed shape of the coupling beam, as shown in Fig. 6.23, suggests that after diagonal cracking considerable deformations would occur also in the transverse direction. This was also confirmed by the fact that the stirrup strains became large as the failure load was approached. The elastic stirrup strains extend over a large enough a height of the beam to suggest that the transverse expansion of the beam may assume, relative to other type of deformations, significant proportions.

To assess the order of magnitude of this type of deformation, the total elongation of the stirrups between the outermost gauge points was determined. In Beam 311 the distance between these points was 20 in., i.e. 70% of the total stirrup length. The transverse expansion of the beam between its edges would be more than what is indicated by these measurements. This is particularly so near the supports where the regions of the largest stirrup strains could not be measured.

The transverse expansion along the beam is shown in Fig. 6.24 for a number of load levels. The dotted line indicates the permanent deformations after 87% of the ultimate load was removed. The regular pattern confirms the symmetrical behaviour. The total expansion over the full depth of the beam at stirrups 5 and 9 may well be twice as much as what is indicated on the diagram. The picture, Fig. 6.24, confirms the positions of the top and bottom edges relative to each other, as indicated in Fig. 6.23. The transverse expansion at midspan, shortly before failure, could have been approximately 0.10 in.

The load-transverse expansion relationship at mid-span, Fig. 6.25, indicates that very large transverse deformations occurred during the last stages of the loading. This diagram very distinctly shows the diagonal cracking load. It appears that this is a more reliable way of defining diagonal cracking than the customary practice of visual inspection. At $P_i = .46P_u$ no cracks could be observed to cross the centre line of the beam, but three cracks appeared while the load was being increased to $.58P_u$.

6.1.6.4 Deflections.

The results of deflection measurements, taken at 6 in. centres along the soffit of the beam proper and at the inner corners of the end-blocks, are shown in Fig. 6.26. From the rotation measurements made on the end-blocks the vertical movement of any point on these "infinitely rigid" blocks could be computed. On the right hand support of the beam a good agreement was found between these independently determined quantities of deflections at all load levels. On the left hand support considerable discrepancy was observed between the block and beam deflections. (See dotted lines.) The vertical movements of the lower left hand corner of the beam are, however, affected by the extensive diagonal cracks (Fig. 6.4) in this area.

In earlier experiments the complete lack of symmetry in the deflection curves for the beam proper was responsible for much concern. It was only later, that it was realised that the transverse expansion of the beam could have been significant and of similar order of magnitude as were the deflections.

Measurements taken along the bottom edge of a deep beam do not represent its deformed shape. If a deflected shape, in the terminology of slender flexural members, can be defined at all for coupling beams, then it must be related to the horizontal axis of the beam. It would then be possible to make a qualitative comparison with deformed shapes of other, i.e. normal, beams.

To be able to estimate the deformed shapes of the coupling beam's axis it was necessary to assume that, conforming with antisymmetry, the top edge of the beam deformed the same way as did the bottom edge. This was drawn and the axis was fitted in between the two edges of the beam. The shape of the axis in each half of the beam was assumed to be predominated by the deformation of the triangle (See Fig. 6.19) in which it was situated. The results of this study are shown for three load levels in Fig. 6.27, in which the dotted lines indicate the displacements of the beam's edges and the heavy lines show the estimated positions of the axis. At the vertical centre line of the beam the displacements of the edges relative to each other are somewhat larger than the corresponding transverse expansions presented in Fig. 6.24. This was to be expected because transverse expansion measurements covered only 65% of the beam's depth.

The "kinks" in the axis of the beam at the supports of the beam proper indicate that shear deformations predominate over flexural deformations.

6.1.7 Crack Formation and Crack Width

As expected, the first cracks formed at the tension corners of the beam and propagated later in a near vertical direction. The diagonal cracks developed gradually from flexural cracks. Only at $P_i = .70 P_u$ was the main diagonal crack formed. The holes around the studs for gauge points on the reinforcement and the stirrups acted as crack initiators. This can be seen in Fig. 6.4. The horizontal cracks, which propagated into the end-block at the tension corners of the beam at the level of the # 8 bars, suggest the presence of high bond stresses. These cracks were only a few inches long. It appears that the 3 feet anchorage for the flexural reinforcement was ample.

To complete the study of this beam the width of some cracks were also measured. The results can be seen in Fig. 6.28. It may be noted that:

a.) The widest crack is the one along the main diagonal. This is to be expected from previous observations. The width of this crack accounts for nearly 70% of the measured transverse expansion at midspan immediately before failure.

b.) The minor diagonal cracks do not increase proportionally with the load but maintain a nearly constant width over a large range of loading. This is consistent with the strain measurements on the stirrups, as shown in Fig. 6.13.

c.) The width of the steep diagonal crack at the upper right hand corner of the beam decreased when the load was approaching its ultimate value. This is consistent with the previously (6.1.4.2) recognised enlargement of the compression area with increased load. See Fig. 6.18.

6.2 Beam 312

6.2.1 Loading and Test Procedure

This beam was similar to the previously described Beam 311, except that:

a.) The content of the web reinforcement was increased. # 4 stirrups were used.

b.) Alternating near-ultimate loading was applied to the beam in six cycles.

In connection with these tests a "loading cycle" is defined as a load sequence in which the load is increased by increments from zero to its maximum and then reduced to zero. A new cycle commences when the same sequence is followed, usually with the load being reversed.

The first load cycle was applied the same way as in the previous test. This load applies anticlockwise moments to both ends of the beam, when the latter is viewed from the East. For the sake of identifying the sense of the loading this cycle is termed "positive". Thus the crack pattern previously discussed (Beam 311) corresponds with positive loading. The reversed load is considered "negative".

"Near-ultimate" load defines a load intensity which caused the flexural reinforcement to yield. At this increment the oil pressure was applied carefully, while the steel strains were observed with mechanical strain gauges. After yielding was clearly observed on both sides of the beam, the load was not increased any further. Readings of the instruments commenced only after yield or creep deformations have visibly ceased.

Four days were required to complete the test. The relevant data on the loading sequence are summarised in Table 6.II.

The procedure of testing was essentially the same as for Beam 311 and for this reason only new features are discussed here.

6.2.2 <u>The Behaviour of the Flexural Reinforcement</u>

6.2.2.1 The distribution of strains.

The distribution of the strains along both layers of the top and bottom reinforcement are presented for the first cycle of the loading in Fig. 6.29. At the last increment the bars in both layers have extensively yielded near the supports.

The strains in the # 8 and # 7, situated in the "compression zone" of the beam also differ in this beam very considerably. The cause of this phenomenon was discussed in 6.1.5 and was illustrated in Fig. 6.19. The larger stirrups restricted, to some extent, the opening of the main diagonal crack and thus the strain difference between the two layers of bars was smaller than in Beam 311.

6.2.2.2 The tension force distribution.

From the mean strains of the four bars of the top and bottom reinforcement the tensile force was computed and then plotted in Fig. 6.30. The pattern is similar to that found for the previous beam, except for the extent of yielding. It was not restricted to the support section only, but extended over a length of 8 to 10 inches.

The characteristic shape of these curves, which so markedly differ from that predicted by conventional beam theory, (shown dotted in Fig. 6.30) is more fully discussed in the summary at the end of this chapter.

6.2.2.3 The position of the internal forces.

From the tension force diagrams the total internal tension was computed at several sections for the four increments of the first load cycle. This is presented in Fig. 6.31. The lowest curve reflects the true beam behaviour, where the magnitude of the internal forces varies proportionally with the external moment. At high loads, however, the tension force again attained, in a rather symmetrical fashion, its maximum value at midspan, as in Beam 311. (Compare this with Fig. 6.8).

Using this information the internal forces were located. For the three high load levels they are shown in Fig. 6.32. The lines of thrusts and the lines indicating the position of the tension resultant lie close to each other, except in the immediate vicinity of the supports. This suggests again that the major change in the mode of the internal load resistance occurs when diagonal cracks develop.

At the sections of maximum moments the gradual decrease of the internal lever arm, z, is evident. Fig. 6.33 shows this reduction in terms of the theoretical internal lever arm. The high computed values of z for low loads result from the tensile resistance of the concrete, which could not be assessed in the experiments.

The crack pattern associated with the steel strains generated in the first cycle of loading may be seen in Fig. 6.34 and Fig. 6.35.

6.2.2.4 The effect of cyclic loading upon the flexural reinforcement.

After the first excursion into the postelastic range it was difficult to assess the magnitude of the steel stresses, and consequent internal forces, with a satisfactory degree of accuracy. There are a number of reasons for this.

a.) The steel, which yielded at the end of a loading cycle, was subjected to relatively large compression immediately after the commencement of the reversed loading of the new cycle. This occurred because the concrete was extensively cracked and was thus incapable of resisting compression forces until these cracks have closed. Therefore most of the compression, required for internal moment resistance at the support sections, had to be supplied at early stages of the reversed load, by the reinforcement. Because of the Bauschinger Effect the stress-strain relationship for the steel becomes non-linear already under moderate-loading.

b.) The source of another difficulty, in assessing the magnitude of the steel forces at the critical sections, arises from excessive cracking in these regions. This is likely to affect the bond performance of the bars. It is not so much the slip of a bar, but rather its movement together with a small lump of concrete, bounded by a number of cracks, which is likely to render the bar ineffective in transmitting substantial forces to the surrounding mass of concrete. Often, after yielding, a strain difference of up to 2000 microstrains was observed between two identical bars of the same layer at the same gauge location.

c.) Residual strains, particularly at the critical sections of the beam were difficult to relate to a residual stress level because the yield strains could increase between the time when the strain measurement was made and the time of the removal of the load. Slow creep was often observed in all deformations at near-ultimate load levels.

These aspects of the behaviour of the reinforcement discouraged any attempt to determine the tensile forces in the elasto-plastic range after a reversal of near ultimate load intensity.

The steel strains in the centre portion of the beam, in the vicinity of the "point of contraflexure" remained largely elastic. Therefore the steel forces could be determined. They are presented in Fig. 6.36. In the left hand side of the diagrams the steel force distributions for the positive, odd numbered, load cycles are shown for three load levels. On the right hand side of the figure the effect of the negative, even numbered, cycles is recorded. In all cases the mean strain of four bars was considered.

It is evident from these curves that there was a large increase in the tensile force intensity after the load has been reapplied the first time in the same direction. This is particularly so for the lowest load increment, i.e. when $P_i = .22\ P_u^*$. Thus there was a large increase in steel stresses from the 1st to the 3rd and from the 2nd to the 4th cycles. For subsequent repetitions of the loadings the stresses generally increase by only a small amount. Had the load not been taken up to near-ultimate intensity at the end of each cycle, it is probable that for a given load the stresses in this region would have remained sensibly constant for a large number of load repetitions. The gradual increase of strains in this test indicated the degree of deterioration, which resulted from the reversals of high intensity loading.

The complete load-strain relationships, for all bars at the centre of the beam, are reproduced in Fig. 6.37. They show a near perfect linear elastic behaviour for all but the first two cycles. In the first load cycle the sharp kink indicates the transition from the uncracked into the cracked state.

On load removal, surprisingly, large residual strains remained in all bars. The likely reason for this is the inability of the diagonal and the flexural cracks to close. Along the diagonal cracks, particularly the major one, shear displacement occurs. Consequently, when the load is removed, the rugged faces of the cracks do not fully fit into each other. Protruding parts of the interlocking small aggregate particles and later debris, which collects in the large cracks, may keep the two faces of a crack apart.

At the commencement of the second load cycle the width of the diagonal cracks is reduced because they cross the line of diagonal compression. Consequently the residual steel strains are also reduced until a new crack forms owing to the negative loading. This second cracking load was always less than that observed in the first load cycle. After the second set of diagonal and flexural cracks were fully formed a near symmetrical elastic behaviour could be observed. (See Fig. 6.37). With two exceptions only little deterioration is noticeable. The inner layer of, # 7, bars show a greater degree of stress increase for negative cycles at the bottom and for positive cycles at the top of the beam. At the last cycle of loading these inner, # 7, bars yielded at gauge points 12 and 33. Presumably prior to failure yielding in these bars has spread from the support to the centre of the beam.

Fig. 6.38 shows the strain history of the two layers of bottom reinforcement at the right hand support of Beam 312. The yield strain was attained at the end of each positive cycle. The first one imposed rather large permanent deformations. The negative cycles probably impose small compression on the outer, # 8, bars and tension upon the inner, # 7, bars. This agrees with the previously observed discontinuity in this locality. (See Fig. 6.19.c). In the last cycle of the loading, immediately before failure, both layers in the "compression zone" of the beam have suddenly been subject to large tensile strains. The behaviour of the corresponding reinforcement at the top of the same section is illustrated by its load-strain history in Fig. 6.39. The first, positive load cycle imposes compression, which, after the formation of the main diagonal crack, changes into tension. During the negative cycles the behaviour of both layers is similar, apart from the different extents of yielding and consequent residual strains. The difference in the strain history owing to the "hinge effect" at the "compression corner" (Fig. 6.19.c), is also evident.

6.2.3 The Behaviour of Stirrups (# 4)

6.2.3.1 The load-stress relationship in the first load cycle.

As in the previous beam, the strains along the central 20 in. lengths of five out of nine stirrups have been measured. The load-stress relationships for four stirrups are reproduced in Fig. 6.40. The heavy lines indicate the gauge localities at which the main diagonal crack crossed the stirrups. In the upper left hand corner of this figure a key diagram shows the position of the instrumented stirrups, the gauge points and also the main diagonal crack which has formed during the first (positive) cycle of loading. This crack just passes beyond the instrumented portion of the outer stirrups.

The first stirrup began to yield at bout 70% of the theoretical ultimate load. With increased load this was followed by yielding of the other stirrup in the middle two third of the beam. Up till the end of the first loading cycle, when 92% of the theoretical load was applied, no yielding was observed in the stirrups adjacent to the supports. However these stirrups too may have been near yielding outside the instrumented lengths.

As only four load increments were used in the first cycle it was not possible to determine clearly the diagonal cracking load.

The straight line, for each of the four stirrups described by Fig. 6.40, indicates an interpretation of the A C I Code. It is to be noted that for each of the three inner instrumented stirrups yield occurred at a lower load than that indicated by the A C I equation. (See 6.1.3.2). The mean strain for the whole stirrup agrees well with the A C I requirements. This finding indicates the importance of using small gauge lengths over the maximum possible extent of the instrumented stirrups.

6.2.3.2 The strain distribution along stirrups in the first load cycle.

The strain distributions along four stirrups are shown in Fig. 6.41 for three increments of the first load cycle. They confirm the previously observed pattern. The yield is first attained at a point where the major diagonal crack crosses the stirrup. The approximate position of these cracks is also sketched in the diagram.

6.2.3.3 The strength of the web reinforcement.

The largest forces, which were developed across the main diagonal crack in each pair of stirrup legs at different increments of the first load cycle, are plotted in Fig. 6.42. By assuming that stresses in the intermediate stirrups, which have not been instrumented, can be estimated from linear interpolation in this diagram, the total shear force resisted by the web reinforcement across the main diagonal crack can be assessed. The total steel force is plotted against the applied load in Fig. 6.43. The shaded area represents the resistance of the stirrups and the remainder of the triangular area represents the contribution of the concrete mechanisms. At approximately 70% of the theoretical ultimate load the web reinforcement appears to have carried all the shear forces. The curve may be compared with that derived from Beam 311 and shown in Fig. 6.16.

6.2.3.4 The behaviour of stirrups during cycle loading.

As was the case with the flexural reinforcement, the behaviour of the stirrups during a reversal of the loading was more difficult to observe. Particularly at the major diagonal crack, where yielding occurred, it was not possible to assess with sufficient accuracy the stirrup force sustained during subsequent cycles. For this reason the load-strain (not stress) history is presented for three typical stirrups at two or three significant gauge points. (For the location of these refer to the key diagram of Fig. 6.40).

To enable the behaviour of the stirrups during cyclic loading to be better appreciated the crack pattern of the beam should be examined first. Fig. 6.44 shows the beam after the first increment of the second load cycle. The new cracks began to cross the others at the lower left and upper right hand corners. During the subsequent increment numerous other diagonal cracks crossed the beam. Very few new cracks developed, however, in the following cycles because additional load reversals caused existing cracks to close or open, according to their orientation. Fig. 6.45 shows the beam at the last increment of the 5th cycle. Apart from a number of secondary cracks this was how the beam appeared already at the end of the 2nd cycle.

Fig. 6.46 illustrates the behaviour of the central stirrup (No.7) Gauge point 73 is crossed by the main diagonal crack extending from the upper left to the lower right hand corner. Accordingly the stirrup at this gauge point is mainly affected by negative loading. (Full lines.) The opposite holds for the other (74) gauge point (Dotted lines.) The difference is particularly demonstrated in the first two cycles. Yield occurred in the stirrup at the end of each cycle at least at one but sometimes at both of the central gauge points. The accumulated residual strains after five load cycles are quite considerable, i.e. 5,000 to 10,000 microstrains.

The strain history for stirrup No.8, situated approximately at quarter span, is presented in Fig. 6.47. The major diagonal crack formed by positive loading crossed the stirrup at point 82. The critical point for negative loading is at gauge 85. However already during the second cycle yield has also set in at the centre of the stirrup, at gauge point 84. At the beginning of some cycles the stirrup was subject to compression but with increased load tension again predominated at each of the gauge points.

Because of the absence of Bauschinger Effects, these load-strain curves can be better related to stresses than similar curves for the flexural reinforcement.

The behaviour of stirrup No.9, situated at the boundary of the beam, is presented in Fig. 6.48. Strains at the top and the bottom of the stirrup (92 and 96) may be compared with those induced at its centre (94). It is interesting to note that after two cycles of near ultimate loading this stirrup has also yielded at midheight of the beam where it was crossed by a steep secondary diagonal crack. (See Fig. 6.45). This suggests that considerable movement in the vertical direction must have occurred at this steep crack.

To enable the gradual increase of the strains along the stirrups to be observed as the loading progressed, the strain distribution for four stirrups is plotted in Fig. 6.49 and Fig. 6.50. These strains occurred at the end of each cycle when the load intensity was of the order of $\pm 86\%$ of its theoretical ultimate value. The location of the major diagonal cracks, which are primarily responsible for the yielding of the stirrups, are also sketched across the base line of each diagram. It is interesting to note that, because of the absence of a crack, at gauge location 64, strains remained elastic throughout the test. The same phenomenon occurred at gauge point 95. These small crack free areas are clearly visible on Fig. 6.45.

6.2.4 Concrete Strains

In the previous test a very large number of gauge points became useless because cracks interfered. For this reason no gauges were provided for concrete strain measurements during the first cycle of the loading. However, after the cracks had fully developed at the end of the first cycle, gauge points were attached to the concrete between the cracks at inclinations which were approximately parallel to the axis of the compression struts formed between two cracks. The gauge points can be seen when Fig. 6.35 and Fig. 6.44 are compared. The four load increments of the first cycle were then reapplied and the resulting concrete compression strains were recorded.

The mean strain obtained from readings on both sides of the beam are presented for five sections across the beam in Fig. 6.51. Because of the random nature of the crack formation, the strain pattern does not conform well with the antisymmetrical load pattern. At the upper right hand corner, where the diagonal compression strut is narrower than at the diagonally opposite corner, the strains are approximately 50% higher. The stresses which correspond to these strains are also recorded at significant points. At the most highly stressed diagonal gauge the compression stress was approximately 87% of the cylinder strength when 92% of the theoretical ultimate load was applied.

At both sides of the vertical centre line of the beam near uniform diagonal compression strains were generated. At the highest load level the corresponding stresses were in excess of 2,000 p.s.i. in this region. The line of thrust, as computed from steel forces, Fig. 6.32, is also shown in Fig. 6.51 for one load increment.

Because the load was applied to a beam, which was already cracked, the load-strain relationship is nearly linear. For a number of gauge points the "load-diagonal compression strain" relationship is shown in Fig. 6.52.

It may be shown from first principles that the diagonal compression stresses in a cracked beam are twice as large as the average shear stresses, v, if the computation is based on the truss analogy of vertical stirrups and 45° compression struts. This theoretical relationship is also shown, by the dotted line, in Fig. 6.52. Leonhardt and Walther[63] have closely examined the intensity of diagonal compression in the web of normal beams. They found that these stresses were invariably in excess of 2v. It is believed that in the statically highly indeterminate lattice the concrete struts, being considerably stiffer than the stirrups, attract more load. Mayer[64] found the corollary to this by observing in his test beams that heavier stirrups accepted proportionally higher forces in otherwise identical beams when subjected to the same load. This observation could not be confirmed with certainty in this test series.

A reasonable agreement exists with this simple theory only at the midspan section of Beam 312. In Chapter 9 an explanation is offered by showing that the analogous truss of coupling beams radically differs from Morsch's model.

6.2.5 Deformations

6.2.5.1 Rotations.

From measurements of the end-block displacements the rotations of plane sections passing through the boundaries of the coupling beams were computed as described previously.

From the load-rotation relationship obtained for the three loading cycles in each direction, as shown in Fig. 6.53, the following observations may be made:

a.) The curves indicate a reasonably symmetrical behaviour, i.e. the rotations at the left, Φ_L, and right hand, Φ_R, of the beam are very similar. The differences originate mainly from the somewhat larger yield of the flexural reinforcement at the right hand support, near ultimate load. It was pointed out earlier that these plastic deformations are extremely sensitive to small variations of material properties, beam geometry or eccentricity of the load.

b.) With subsequent load cycles the right hand support shows a more pronounced deterioration. This is indicated by a larger loss of stiffness at that end and by larger yield rotations.

c.) During the first cycle of loading, the onset of cracking and the yielding of the reinforcement can clearly be observed.

d.) A marked reduction of the stiffness may be observed immediately after the commencement of the loading in the second cycle, i.e. reversed loading. This is due to the fact that flexural and diagonal cracks, which developed during the previous load cycle, need first be closed before flexural or diagonal compression stresses can be generated in the beam.

e.) The elastic recovery upon load removal, as indicated by the near parallel straight lines, shows that considerable permanent deformations occurred already after the first and second cycles of loading. It was impossible to separate the permanent distortions caused by flexure (yielding of the main reinforcement and plastic concrete deformations in the compression zone), or by shear (yielding of the stirrups and distortions of the diagonal concrete struts), and by bond (slips in the body of the beam and possibly at the anchorages).

f.) An unusual feature of the load-rotation curves for all subsequent load cycles is the very gently sloping portion between zero load and approximately 40 Kips. This indicates that very large deformations occur immediately after the commencement of the loading. The flexural and diagonal cracks are being closed only gradually and a subsequent "hardening" of the beam can be observed. Between approximately 30% and 75% of the theoretical ultimate load a linear elastic behaviour becomes apparent. In this range the loading and unloading curves (3rd to 5th cycles) are approximately parallel. In the last (6th) cycle of the loading very large rotations occurred before the beam could significantly engage in load resistance. The large diagonal cracks, i.e. stirrup strains, were primarily responsible for this behaviour.

Unfortunately, the large yield rotations in the final cycle, prior to failure, could not be followed because of the hydraulic loading system.

6.2.5.2 The variation of stiffness.

From the changing slope of the load-rotation relationships, shown in Fig. 6.53, the gradual loss of the beam's stiffness is evident. This changing of the stiffness with the progress of loading is quantitatively expressed more clearly in Fig. 6.54.

The curve, indicating the variation of stiffness during the first load cycle (marked 1), shows that after cracking the stiffness is abruptly reduced to about one fifth of its original value. It is to be noted that the experimental determination of the stiffness of the uncracked section is uncertain in this particular test. In the first increment the load was brought to a level where a few flexural cracks have already appeared. (Fig. 6.34). Several increments would have been required at lower loads to determine the stiffness in this range more accurately.

It was mentioned in 6.1.6.1 that a good agreement exists between the measured values of the stiffness of the uncracked beam and the theoretical ones. This theoretical load-rotation relationship, shown also by a straight line in Fig. 6.53, was used to define the stiffness of the uncracked beam in Fig. 6.54.

The reduced stiffness of the cracked beam was maintained during the first load cycle until the load reached approximately 75% of the theoretical ultimate intensity. After this, with the onset of yielding, the stiffness rapidly diminished. The stiffness indicated by the unloading at the end of the cycle is shown dotted.

Curve 2 shows an immediate reduction of stiffness upon load reversal until a steady value is attained.

During the subsequent cycles Fig. 6.54 indicates three distinct load ranges in which the stiffness varies in different ways.

a.) A "hardening" range when the stiffness increases. This is associated with the closure of cracks.

b.) A "steady" range where the stiffness is approximately constant. This represents linear elastic behaviour.

c.) A "softening" range where the stiffness is observed to decrease. Plastic concrete deformations and yielding of the reinforcement is responsible for this.

Generally the stiffness was found to gradually decrease with each repetition of loading. The same observation applies to the stiffness associated with unloading, as shown by the dotted lines in Fig. 6.54.

The stiffness of the beam during positive loading (odd numbered curves) is somewhat larger than during the negative cycles of loading (even numbered curves). This is likely to be due to the formation of the cracks during the second cycle of the loading, which was influenced by the existing set of cracks formed during the first load cycle.

Fig. 6.54, which is based on the average rotation of the two ends of the beam, gives a good picture of the effect of high intensity cyclic loading upon the rate of deterioration in coupling beams.

6.2.5.3 The elongation of the beam.

During the loading in both directions the end-blocks of the test specimen gradually moved further apart. It will be shown later that the major part of this movement is due to the elastic and plastic elongation of the beam proper. The elongation increases with the extension of the top and bottom reinforcement, which was generally found to be subjected to tension over the whole length of the beam.

The behaviour and properties of Beams 311, 312 and 313 are very similar. For this reason the load-beam elongation relationships for all three beams are presented together in Fig. 6.55. The curves for Beam 312 (full line) show clearly that at the end of each cycle a permanent elongation was imposed upon the beam. With the exception of the first load cycle this permanent deformation was approximately equal to the yield that occurred at the end of a particular cycle. The stages of cracking and onset of yielding identically appear in all 1st cycle curves. Immediately before failure the elongation in Beam 312 was approximately $\frac{1}{4}$ inch.

6.2.5.4 Transverse expansion.

In Beam 311 the transverse expansion was assessed from the elongation of the stirrups over 20 in length only. In this beam two additional gauge points were provided from steel to concrete, at each instrumented stirrup. This enabled the transverse expansion to be determined over 28 in length, i.e. more than 90% of the overall height of the beam. Practically all the transverse strains occur within this depth.

Measurements have been made at all load increments, but they are reproduced in Fig. 6.56 for the 1st, 2nd and the 6th cycles of the loading only.

The symmetrical distortions during the first cycle of loading are apparent. They are very similar to those obtained for the previous beam (311). However, they are smaller because of the presence of heavier web reinforcement.

In the second cycle of loading it is noticeable that the beam deteriorated more in the right hand half of the span. This is also evident from the observed strain distribution along stirrups 8 and 9, shown in Fig. 6.50. The very large transverse expansion (over ¼ in.) near the support was measured in the last (6th) cycle of the loading. Fig. 6.57 shows this half of the beam after failure.

The progress of transverse expansions at stirrups Nos. 5, 7 and 9 are shown for the whole loading sequence in Fig. 6.58.

6.2.5.5 Deflections.

The displacement of the bottom edge of the beam was measured with dial gauges, as described previously. The results are shown in Fig. 6.59 for four increments of the first two load cycles. The rotations of the end blocks were determined independently.

The problems associated with the determination of the deformed axis of the beam were discussed in connection with Beam 311. In this beam no assumption was made with respect to the position of the top edge. The transverse expansions, obtained over 90% of the depth of the beam, were simply added to the line representing the deformed soffit of the beam. Thus the outline of the top and bottom edges were obtained as shown in Fig. 6.60. Only the highest load increments for the first two cycles are shown in this diagram. The commencement of the deterioration of the right hand half of the beam at the end of the second cycle is noticeable. The estimated shape of the deformed axis is shown by the full line. Considerations which were discussed in connection with Beam 311 formed also the basis of the construction.

The pronounced shear deformations of the coupling beam are evident from this diagram (Fig. 6.60).

6.2.6 The Failure Mechanism

The behaviour of Beams 312 and 313, both subjected to cyclic loading, is very similar. The details of the failure mechanism, which are distinctly different from those observed in Beam 311, are discussed later in 6.3.5.

6.3 Beam 313

6.3.1 Loading and Testing Procedure

The third beam of this series differed from the previous ones in two respects;

a.) the stirrup size was increased to # 5

b.) the concrete strength was unintentionally higher.

It was intended to repeat the load program as applied to Beam 312 with the exception that after the third cycle the load would have had to be slightly increased so as to impose upon the beam large flexural plastic rotations. It was intended to observe the effect of large flexural yielding upon shear strength in the presence of heavy web reinforcement.

The load, which did produce large yield strains in the flexural reinforcement, was maintained for some 20 minutes at the end of which all dial gauges and spot readings of Demec gauges became nearly stable. Then suddenly yielding commenced again and accelerated. Before the oil pressure could be reduced the beam failed. The three cycles of loading however did furnish some useful information.

The relevant data for this beam and its loading sequence are summarised in Table 6.III.

6.3.2 The Behaviour of the Flexural Reinforcement

6.3.2.1 The distribution of strains.

The distribution of strains for both layers of the top and bottom reinforcement are shown for all three cycles of the loading in Fig. 6.61, Fig. 6.62, and Fig. 6.63. The first of these, showing the strain distribution in the first cycle of the loading, enables a comparison to be made with the previously described beams. The strain patterns for all beams of this series are also compared in the summary, at the end of this chapter, (6.5).

It is noticeable that at higher load levels and after the reversal of the loading the inner layer of bars (# 7) show a more unsteady pattern than the bars of the outer layer (# 8). The points of low strain readings for the inner (# 7) bars happen to coincide with those gauge lengths at the centre of which a welded stirrup stud is located.

In the second, but particularly in the third cycle of loading, the strains at the inner layer of (# 7) reinforcement became considerably higher at each load increment. At the maximum load of each cycle the flexural tension reinforcement yielded at the supports. The resulting permanent strains were considerable. This is also apparent in Fig. 6.63 where the permanent tensile strains in the compression zones of the beam are indicated.

The large permanent tensile strains observed in the compression zones of the beam, after load reversal, would suggest that the cracks, previously formed, could not fully close in the vicinity of these yielded bars. Therefore one would expect that the flexural reinforcement would have to carry considerable compression forces.

The strain history of the flexural reinforcement at the right hand support of the coupling beam indicates that at gauge points 23 and 24 small compression forces were indeed present during the second load cycle. This may be seen in Fig. 6.64. Because of Bauschinger Effects it is difficult to assess the magnitude of these compression forces. Observation of the beam showed however that even the widest cracks, which formed during the first cycle of the loading, have closed during the second cycle. Indeed they must have closed otherwise the large (diagonal) compression forces could not have been transmitted.

6.3.2.2 The position of the internal forces.

It is probable that, because of the large tensile strains imposed upon the reinforcement during the last load increment of the first cycle ($P_i = .93P_u^*$), the concrete surrounding these yielded bars could not effectively resist compression in the second load cycle. After the closure of the cracks the concrete in the immediate vicinity of the yielded bars was probably subject to small compression strains only. Thus the compression strain distribution across a support section during the second load cycle is likely to be different from the pattern observed on the previous beams. (See Fig. 6.18 and Fig. 6.51).

The centre of the compression at the supports is likely to move towards the centre of the beam where yielded reinforcement does not interfere with the development of concrete compression strains. The probable situation is expressed qualitatively in Fig. 6.65. The consequence of the movement of the compression resultant, C' , is a smaller internal lever arm, z_2 .

The top diagram of Fig. 6.62 may be considered as evidence for this phenomenon. The tensile (top) reinforcement near the right hand support was subject, in the second load cycle, to considerably higher strains than to what it would have been during a first cycle loading of the same sense. The reinforcement, at gauges 21 and 22, began to yield when the load was only 74% of the theoretical ultimate. At this load intensity stresses at the equivalent locality during the first cycle were of the order of 36,000 p.s.i. only.

6.3.2.3 Load-strain relationship.

Fig. 6.66 compares the load-strain relationship of the tensile reinforcement at the supports during the first and second cycles of the loading. In all cases the mean strain for the four bars was considered. The load-strain curve for gauge points 1 and 2 runs close to the theoretical line (shown dotted), which was based on a constant internal lever arm of 25 inches. After the first load cycle this steel showed a mean permanent set of 3400 microstrains upon load removal. This affected the surrounding concrete in the second cycle and contributed towards a reduction of the internal lever arm. Therefore the steel, which operated on this reduced internal lever arm at gauges 43 and 44, was subject to higher strains, as indicated by the heavy dotted line in Fig. 6.66.

Similarly yielding of the reinforcement in the first cycle, at gauge locations 23 and 24, has affected the performance of the steel at gauges 21 and 22 during the second cycle of the loading. The effect in this case was much larger because the permanent mean set at the end of the first load cycle was 8800 microstrains at gauge points 23 and 24. (Also see Fig. 6.64).

The cracks, to which reference was made previously, may be examined on the photographs which show this beam at various stages of the loading. See Fig. 6.67, Fig. 6.68, Fig. 6.69 and Fig. 6.70.

6.3.3 The Behaviour of the Stirrups (# 5)

6.3.3.1 The load-stress relationship in the first load cycle.

In order to enable stirrup strains to be measured also in the immediate vicinity of the flexural reinforcement, the gauge points in this beam were slightly rearranged. Six four inch gauge lengths were provided, instead of five, at both sides of the beam for every second stirrup. This is shown in a key diagram at the upper left hand corner of Fig. 6.71.

From the load-stress relationship for four stirrups shown in this figure it is evident that during the first cycle of loading all stirrups performed in a similar manner, and that the stresses remained well within the elastic limit. The four graphs, shown for each stirrup, represent the gauge lengths which were stressed most highly. They also include the gauge at which the major diagonal crack crossed the stirrup. The stresses at these latter gauges are shown by the heavy lines. Contrary to the observations made on the previous beams, stresses at these distinguished points do not exhibit particular features. This suggests that, with the generous provision of web reinforcement, the failure mechanism changes.

It is also apparent that the stresses vary considerably along the stirrups. Surprisingly, however, the mean stress lies close to the line which represents the A C I relationship. Some gauge lengths were crossed by diagonal cracks at the second load increment, some others were crossed only after the third increment. The increments were too far apart to allow the diagonal cracking load to be determined more accurately. From the point of view of this investigation, however, the diagonal cracking load has no particular significance.

6.3.3.2 The strength of the web reinforcement.

For the sake of a comparison the performance of the stirrups, at points where these crossed the major diagonal crack, has also been studied. The forces resisted by each pair of instrumented stirrup legs are plotted in Fig. 6.72. A rather regular pattern ensues, which is consistent with the previously described behaviour of the two triangular halves of the beam. (Fig. 6.19). The contribution of all stirrups crossing the main diagonal is plotted against the load in Fig. 6.73. From this it may be seen that over about 125 K. applied load these stirrups resist an equal or larger force.

Because the strength of all stirrups is larger than the possible maximum external load, the failure mechanism, associated with the separation of the beam along the major diagonal, can not develop.

The curve suggests a diagonal cracking load of approximately 80 K (v_{dc} = 476 p.s.i.)

6.3.3.3 The behaviour of stirrups during cyclic loading.

The full strain history of each of the instrumented stirrups during the three cycles of loading is shown in Fig. 6.74 and Fig. 6.75. For the sake of brevity, only the most highly stressed gauge lengths for each of the stirrups is reproduced.

A comparison of the behaviour of the five stirrups shown in these two figures shows clearly that the beam deteriorated more rapidly towards the right hand support.

In the left hand half of the beam the stirrups performed in the elastic range during all three load cycles.

Residual stirrup stresses, after the removal of the loading, remained at the level of 7500 p.s.i. The corresponding compression forces in the concrete must have been tranmitted across the cracks by interlocking aggregate particles. It was frequently observed, particularly at high load intensities, that a shear displacement occurred. This is a movement of one face of a crack relative to the other in the direction of the crack. Upon load removal the two rugged forces of a crack could not fully fit into each other because of permanent shear displacements. The protruding aggregate particles are likely to prevent the full closure of the crack and are thus responsible for the relatively high stirrup stresses at no load. Fenwick[60] measured such shear displacement along diagonal cracks formed in normal beams. After repeated alternating loading sometimes debris could also be observed in the cracks.

The considerably larger plastic deformations of the flexural reinforcement at the right hand end of the beam (Fig. 6.64) had a marked effect upon the performance of the stirrups in this region. At the end of the second (negative) load cycle, large yielding occurred at the upper right hand corner of the beam. Also shear displacements developed along this wide vertical crack, which joined up with its counterpart formed during the previous load cycle. (See Fig. 6.68). When the third (positive) load cycle was applied, this wide crack across the top of the beam closed again but considerable local crushing and pulverization occurred after the two rugged faces of the crack have met. This area was still capable of transmitting compression forced during the subsequent loading, but it could not transmit the high shearing forces without sliding. Thus the remainder of the beam, situated above and to the left of the two steep diagonal cracks, which run towards this compression corner (see Fig. 6.69 and Fig. 6.70) was permitted to move upwards. This movement immediately engaged a few and clearly insufficient number of stirrups. Consequently they yielded across the steep cracks. The close interplay of the flexural and shear reinforcement is further discussed in connection with the failure mechanism of the beam.

6.3.3.4 The strain distribution along the stirrups.

The strain distributions over the full instrumented length (24 in.) of the stirrups are shown for the last increment of each of the three load cycles in Fig. 6.76. In the left hand half of the beam near uniform stresses were observed along the stirrups. With the use of heavier stirrups (# 5) this was to be expected.

The extent of yielding at the right hand support, at stirrup No. 9, indicates that considerable shear displacements must have occurred across the compression zone of the beam.

6.3.4 <u>Deformations</u>

6.3.4.1 Rotations.

The load-rotation relationship shown for all three load cycles in Fig. 6.77 confirms the previous observation that the right hand end of the beam was subjected to larger plastic deformations. Apart from these different yield deformations, the behaviour of the two ends of the beam was very similar. As a result of the larger yielding of the flexural reinforcement at the right hand support, there was a correspondingly larger loss of stiffness at that end of the beam. The graphs will be further discussed when they are compared with curves of other specimens of this series.

6.3.4.2 The variation of stiffness.

The variation of the mean stiffness of the beam with alternating loading is shown in Fig. 6.78. The dramatic loss of stiffness immediately after the formation of flexural and diagonal cracks is again apparent. With one exception, the features of these curves are similar to those presented for Beam 312 in Fig. 6.54. In this beam there was a continual loss of stiffness right through the second loading cycle. The number of repetitions of the loading were unfortunately insufficient to supply adequate information with respect to the loss of stiffness owing to high intensity alternating loading. It is certain that the stiffness in the steady range during the third cycle was no more than one-sixth of the stiffness of the beam proper in the uncracked state.

6.3.4.3 The elongation of the beam.

The movements of the end-blocks relative to each other, which were defined as the beam's elongation, were very similar to those measured in the previous tests. They are shown in Fig. 6.55. It is evident that the increased amount of web reinforcement had no appreciable effect upon the elongation of the flexural reinforcement.

6.3.4.4 Transverse expansion.

It was observed previously that the overall performance of the web reinforcement can be conveniently studied by measuring its total elongation, i.e. the transverse expansion of the beam. In this beam this was carried out over 24 in. length, which corresponds to 77.5% of the depth of the beam or 86% of the stirrups' length. Fig. 6.79 shows the results for Beam 313. The dotted lines marked .00 indicate the permanent deformations produced after the removal of the last increment of the previous load cycle.

The elastic behaviour during the first two load cycles and the deterioration over the right hand half of the beam during the third one is evident from the three diagrams.

When comparing this figure with other transverse expansion diagrams, it should be noted that a different scale was used.

6.3.4.5 Deflections.

The information on deflections, previously presented in two figures, has been combined into one for this beam. Fig. 6.80 shows the deformations for three stages of the loading during the first cycle, and for two stages of the second load cycle. The lower lines for the beam proper were obtained from dial gauge measurements along the soffit of the beam. The sloping top lines were found by adding the previously determined transverse expansions of the beam to the bottom lines. These two lines give thus the general deformed shape of the beam at various stages of the loading. Between these the estimated position of the deformed axis of the beam is drawn with a heavy line.

A particular feature of this diagram is the discrepancy between the positions of the right hand end of the beam, as determined from measurements on the beam proper, and the positions found from the rotations of the end-block. It was mentioned previously that considerable shear displacements can occur in the compression zone of a beam. This zone was seriously disturbed by a wide crack during the previous load cycle. The phenomenon may also be termed a "shear slip" as indicated on Fig. 6.80. Measurements showed that at the end of the third load cycle this "shear slip" at the right hand support was of the order of $^{1}/10$ in.

It is also interesting to note that the estimated axis of the beam proper is only slightly deformed relative to the overall deformations of the structure. At high load increments i.e. at $P_i = .88$ to $.92 P_u^*$, the yielding of the flexural steel undoubtedly accounts for a considerable portion of the abrupt angular change at the beam - end-block junction. However at the lower increments, i.e. $P_i = .53$ to $.74 P_u^*$, all reinforcement performed elastically and therefore the flexural behaviour of the beam can not account for the angular discontinuities at the supports, which resemble the features of "plastic hinges". It is the predominance of shear deformations in the beam proper which is responsible for this behaviour.

The distortions caused by shear and the effect of shear forces upon the stiffness of coupling beams is discussed in Chapter 9.

6.3.5 The Failure Mechanism

The failures of Beams 312 and 313 clearly differed from that of Beam 311. The latter was separated along the main diagonal into two equal halves because of the failure of the web reinforcement. In this beam, however, the strength of the stirrups, crossing the main diagonal, was well in excess of the flexural capacity of the main reinforcement. It is doubtful whether the load on the beam could have been increased much further at the end of the first cycle. At 93% of the theoretical ultimate the tension steel at both supports had already yielded. The decreased internal lever arm would not have permitted much increase over that (93%) load level.

The effect of load reversal upon the condition of the concrete in the compression zone was pointed out previously. It is believed that the permanent shear displacements produced in one cycle are largely responsible for the destruction of the concrete during a subsequent load cycle, when the cracks passing across the compression zone are being forcibly closed. This destruction of the concrete along near vertical cracks must reduce the shear transfer capacity of an area where high intensity diagonal compression prevails. A few stirrups passing through the compression zone and the dowelling action of the main reinforcement will counteract, to a certain extent, a "shear displacement" or "shear slip". Fig. 6.57 and Fig. 6.70 both indicate that the beam tends to separate along a near vertical crack. Consequently very few, if any, stirrups can assist in shear transfer. The excessive yielding of the instrumented stirrup, adjacent to the support section, was invariably observed in all beams which failed in this manner.

The beam (313) possesses the features of a flexural failure. However, dowel displacements, the yielding of outer stirrups and the visible shear displacements along cracks indicate that the high shear force is primarily responsible for the destruction of the compression zone.

In some beams crushing of the concrete due to sliding, in some others a diagonal compression failure and consequent lateral bursting of the compression zone was more prominent.

6.3.5 The Repaired Beam (1313)

Visual inspection of Beam 313, after its failure, revealed that the area near the left hand support showed no signs of serious distress. Strain measurements made on the reinforcement in this area, and evaluated much later, confirmed this. The stirrups had not yielded and the maximum strain, which occurred prior to failure in the flexural reinforcement, was of the order of 5000 microstrains.

The failure, which occurred at the right hand support, can be seen in Fig. 6.81 to have been restricted to a relatively narrow vertical band of the beam proper.*

It was decided to repair this beam and to reload it to failure. For reference purposes the specimen was labelled BEAM 1313.

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* This picture shows the complete test frame with both 100 Tons capacity jacks in position. Note that the view is from the West.

The beam was left in the test frame and in this position the heavily cracked and crushed concrete was removed until the clean reinforcement was exposed. The close up of the beam at this stage is shown in Fig. 6.82. A slight kink in the flexural reinforcement is noticeable. This resulted from the "shear slip" which occurred along the near vertical failure crack.

The formwork was attached to both faces of the beam by means of clamps and the new concrete was placed through very small gaps left between the top bars, and between the bars and the formwork. Similar difficulties of construction would be encountered if the repair of the coupling beams in a real shear wall structure would have to be attempted.

Necessarily a concrete with a suitable workability was required. The size of the coarse aggregate was $^3/8$ to $^3/16$ in. The water-cement ratio was 0.375, so that, with the use of 2% Calcium Chloride, a cylinder strength at the time of the test - four days after placing this concrete - of 6180 p.s.i. could be attained. This was a little less than the strength of the concrete in the remainder of the beam.

Only the rotations of the end-blocks were measured during the test. The load was brought up in small increments till failure occurred. Surprisingly the failure load (176.0K) was 10% higher than the theoretical ultimate and 22% higher than the previous failure load. The failure occurred at the other, damaged end of the beam. The form of the failure crack was very similar to that observed in the first failure. Lateral bursting of the concrete indicated that high compression stresses existed at the lower left hand corner of the beam. The view of the repaired beam after failure is shown in Fig. 6.83. A close up of the failure section, Fig. 6.84 shows the crushed concrete owing to diagonal compression, and it also shows the "shear slip" that has taken place.

It is necessary to point out that the load was applied in the same positive direction as in the 1st and 3rd cycles of the loading, i.e. in the direction in which the beam first failed. The left hand (cracked) end of the beam was subjected to only one load reversal during which it suffered little damage. The maximum tensile strains in the flexural steel of that end of the compression zone (at gauge points 43 and 44) were only of the order of 2500 microstrains. Moreover the beam has also gained some strength during the 39 days which elapsed between the two tests.

At the right hand side of the repaired beam the flexural reinforcement entered the strain hardening range immediately upon straining to yield strength level. This is evident from the records shown in Fig. 6.64 for gauges 23 and 24. It is almost certain that at failure the reinforcement at the failure side of the beam was also well into the strain hardening range. These increased stresses account for the increased ultimate strength of the beam.

Surprisingly the beam rotations at both ends were almost identical. This may be seen from the load-rotation relationship shown in Fig. 6.85. A small change of stiffness can be observed at the right hand and when the diagonal cracks penetrated the newly placed concrete. The "soft range" which is usually observed at low repeated loads (see Fig. 6.53) is absent.

It need be remembered that the sense of the load on this repaired beam was the same as that of the last load on the unrepaired one. Thus no cracks had to be closed. Few new cracks could form across the repaired end of the beam. This explains the near linear behaviour between zero load and 90% of theoretical ultimate load.

The strain hardening of the reinforcement enabled the load through the jack to be maintained in the plastic range.

Very small increments were used near the failure load. This enabled the large plastic rotations to be determined. It was pointed out earlier that, with the adopted testing arrangement, the beams always "run away" before failure and that this prevented the recording of the full plastic deformations. It is most probable that the plastic rotations, which occurred in Beam 313 before the load dropped off, were in excess of those recorded for the repaired beam 1313 and presented in Fig. 6.85. It may be said that, at least for one way loading, there was appreciable ductility available in Beam 313. Almost all of this originated from flexural deformations.

The elongation of Beam 1313 was recorded, for the sake of comparison, in Fig. 6.55. It shows that the total elongation of the beam after repair was of the same order as the elongation imposed by the previous three cycles of near ultimate load. The beam elongation shows somewhat larger ductility than the load-rotation relationship.

The stiffness of Beam 1313 may also be compared with the original values of the stiffness shown in Fig. 6.78.

6.4 Beam 314

6.4.1 Loading and Testing Procedure

The last beam of this series was in all but one respect identical with the previous one (313). To examine the effect of nominal, horizontal reinforcement, which under usual conditions would be provided by a designer in such a relatively deep beam, two pairs of # 5 bars were provided 8 in. apart and placed symmetrically about the horizontal axis of the beam. It is common practice to make the size of the secondary reinforcement in beams the same as that of the stirrups. The assembled cage, complete with welded studs and protective tubes, can be seen in Fig. 6.86.

Three positive and two negative load cycles were applied. Details of the loading are set out in Table 6.IV. At the last increment of the 5th cycle, when strain readings were about to be taken, yielding set in again, and before the load, which was maintained for 8 minutes, could be removed, the beam failed.

In the computation of the ultimate load, the contribution of the four # 5 bars was included. They increased the theoretical ultimate load computed for the identical previous beam (313) by 18%. It must be realised, however, that even under the usual assumptions of linear strain distribution, extremely large yield strains would be required in the main bars if all the secondary # 5 bars were also to yield. The theoretical ultimate moment is only used as a reference quantity.

Fig. 6.87, Fig. 6.88 show the crack pattern of the beam at two stages of the loading.

6.4.2 <u>The Behaviour of the Flexural Reinforcement</u>

There were no new features revealed in the distribution of the strains in the top and the bottom steel. The pattern was very similar to that observed in Beam 313. The differences between strains in the two layers at the top and the bottom steel were rather small except near the supports. Detailed results of these measuremenrs are not reproduced here, but the mean strains for the four top and four bottom bars are compared with the mean strains obtained in a similar manner for the other beams of this series. These diagrams (Fig. 103 and Fig. 104) are discussed in greater detail in the summary at the end of this chapter.

Because of the contribution of the secondary, # 5, reinforcement to moment resistance the stresses in the main flexural reinforcement are correspondingly lower over the full length of the beam.

The strain history of the flexural reinforcement at the right hand support, where the failure occurred, indicates considerably smaller yielding at the end of the first four cycles than in the previous beams. This is quite apparent when Fig. 6.89 and Fig. 6.64 are compared. The yielding at the left hand support was even smaller. The maximum steel strain there was of the order of 3000 microstrains.

It is considered that the secondary reinforcement, which did not yield till the last increments of the test, prevented the main reinforcement from yielding more extensively. It had a restraining or stabilising effect. The last load increment at the end of a cycle was generally determined by observing the onset of yielding at the critical gauges of the flexural reinforcement. Usually up to 10 minutes were required till the load and displacements were stabilised, i.e. when the dial gauges stopped moving. In this beam this process was much quicker.

The loads applied to this beam at the 4th increment of each cycle were already considerably higher than at any load previously used. Because of the uncertainty about the contribution of the secondary reinforcement, it was considered unwise to increase the load any further for the sake of obtaining larger plastic deformations.

At the end of the 5th cycle readings at gauge points 23 and 24 indicated 16000 and 18000 microstrains respectively. The premature failure of the beam prevented other readings being taken.

6.4.3 The Performance of the Horizontal Web Reinforcement

A few years ago the contribution of horizontal web reinforcement in beams was a much debated issue in Europe. Rausch in his work on shear and torsion[65] presented the view that diagonal principal tension in the web can be resisted only by diagonal bars or by an orthogonal mesh. The latter was thought to be capable of resisting both the horizontal and the vertical components of the diagonal tension. Followers of this school - in which the principles applying to homogeneous, isotropic, elastic solids were transplanted onto reinforced concrete - failed to appreciate that in a diagonally cracked web the diagonal tensile stresses disappeared. After diagonal cracking an entirely new mechanism (so early recognised by Mörsch) is formed, in which vertical stirrups alone can form part of an adequate shear resisting system.

Leonhardt and Walther have found from experimental studies[66] that the horizontal web reinforcement in normal beams was not appreciably stressed. Horizontal bars in the web did not appear to have affected the behaviour of stirrups at all. They held the view that such bars in fact interfere with the free development of the diagonal compression struts of the analogous truss system and thus they reduce the ultimate shear strength of the beam.

It is now a common practice to retain such horizontal web reinforcement in deep beams, largely for the sake of crack control.

The above discussion applies to beams of normal proportions. To the writer's knowledge no beams of such relative dimensions and load pattern as those reported here have previously been studied.

It was pointed out, in 6.1.6.2 that, as a consequence of the load, the length of coupling beams must increase. The strain distribution along both the top and bottom reinforcement, as well as the measurements of the end-block displacements have clearly confirmed this. Therefore it was to be expected that any other reinforcement, placed between and parallel to the main flexural bars, would also have to become longer and thus be subjected to tensile stresses. To confirm this all four # 5 intermediate bars were instrumented over the entire length of the beam proper. (See Fig. 6.86). Strain readings were taken at each load increment.

The results are presented in Fig. 6.90. The top two sets of curves show the strain distribution along both layers of secondary # 5 reinforcement for all four increments of the positive (1st and 3rd) cycles of loading. The strains are approximately uniform and show a slight increase only where the particular layer is situated nearer to the tension zone of the beam at the supports. Strains for the same load intensity in the 3rd cycle are invariably higher than in the 1st cycle, a phenomenon repeatedly observed on the main flexural reinforcement. The bars near the tension zone at the supports attained the yield stress in the 3rd cycle.

The lower two sets of graphs indicate the strain distribution during the negative (2nd and 4th) cycles of the loading, again for both layers of # 5 bars. Only two load levels are shown in this half of the figure. Though less uniform the pattern is essentially the same as the one obtained for the positive cycles.

The measurements thus verify once more the elongation of the coupling beams. Because these # 5 secondary bars are subject to high stresses they do contribute towards the strength of the beam.

The horizontal secondary reinforcement appears to have influenced the crack pattern of the beam in two ways.

a.) In all previous beams the diagonal cracks originated from flexural cracks and propagated gradually into the web. The major diagonal crack was the last one to form. This visual observation was also confirmed by the behaviour of the stirrups. In Beam 313 the high stresses in the stirrups developed last at those points at which the major diagonal crack crossed the stirrups. (See the heavy lines in Fig. 6.71). In this beam, however, the crack along the major diagonal developed earlier, before flexural cracks penetrated into the central portion of the beam. The stress-strain relationship for the stirrups, to be shown later in Fig. 6.91, also confirms this observation.

b.) When the photographs of the two, otherwise identical beams are compared (Fig. 6.67 and Fig. 6.87) it is seen that at the completion of the first loading cycle a considerably finer crack pattern has evolved in Beam 314. This difference also applies to the reversed load cycle. It will be seen that, as a consequence of this finer crack pattern, a more uniform distribution of stresses developed along the stirrups.

6.4.4 The Behaviour of Stirrups

6.4.4.1 The load-stress relationship in the first load cycle.

With respect to the principal reinforcement this beam was identical to beam 313, hence no significant difference in the behaviour of the stirrups was expected. Indeed the differences were small. Apart from the more uniform distribution of the strains along the stirrups the secondary reinforcement did not appear to have influenced the behaviour of the stirrups.

The load-stress relationships shown in Fig. 6.91 for four stirrups indicate a rather good agreement with the current A C I recommendations. In each diagram the four largest of the measured stresses are recorded. The scatter between these is small with one exception. The last stirrup, near the support, behaved differently. An inspection of the crack pattern of the beam at the end of the first load cycle, as shown in Fig. 6.87, reveals that the stirrup crosses the critical diagonal cracks near the top of the beam. The tensile forces in this stirrup are then disposed of gradually, by means of bond, into the adjacent end-block, hence the stresses gradually decrease towards the bottom of the beam.

6.4.4.2 The strength of the web reinforcement.

Once more the possible failure mechanism along the major diagonal crack was examined. The stirrup forces at the points where the stirrups cross this major diagonal have been computed and plotted in Fig. 6.92. The symmetrical behaviour during the first cycle is again evident. From these the total shear force resisted by nine stirrups could be derived and compared with the external load. The results are presented in Fig. 6.93.

The only noticeable difference in the behaviour of the two beams (313 and 314) is, (when Fig. 6.73 and Fig. 6.93 are compared) that a larger portion of the shear force is being resisted in Beam 314 at the early stages of the loading. This is due to the early development of the main diagonal crack. At higher load the contribution of the # 5 stirrups is practically identical in the two beams. The contribution of the stirrups exceeded the external load hence no failure mechanism associated with the major diagonal could develop.

6.4.4.3 The behaviour of stirrups during cyclic loading.

The load-strain history at the most highly stressed gauge point of each instrumented stirrup is shown in Fig. 6.94 for all five load cycles. It is evident that all stirrups performed in the elastic range.

The formation of the first diagonal cracks in the 1st and 2nd cycles of the loading is evidenced by the kink in the corresponding curves. During the 3rd and 4th cycle, when very few and only insignificant cracks developed, all stirrups showed a nearly linear elastic behaviour. The residual strains changed very little from cycle to cycle and were about the same as in Beam 313. ($f_s \approx 7500$ p.s.i). Only a small degree of deterioration is noticeable. The approaching distress of the beam in the 5th cycle at the right hand support (Stirrup No.9) is apparent already at the first two load increments. This is an indication that the compression zone is incapable of transmitting these shear forces without undergoing excessive shear (slip) deformations. This is the only instrumented stirrup which restricts the relative movements of the two parts of the compression zone.

The central stirrups performed approximately the same way during the positive and negative load cycles. The stirrups near the support, on the other hand, reacted differently. The gauge points selected and represented in Fig. 6.94 are those which are significant only during positive loading. When the load is reversed the other ends of the outer stirrups become more highly stressed. This is also shown in the next paragraph.

It is to be noted that after the second cycle of the loading the load-strain relationship lies well away from the line which represents the A C I relationship. Admittedly the A C I recommendation is an ultimate load equation. No claim was made that it also predicts the behaviour of the stirrups over the whole range of the loading. The general tendency of the curves strongly suggests, however, that no relief can be expected from the contribution of the concrete mechanism towards shear resistance after one or two near ultimate load cycles in each direction.

6.4.4.4 The strain distribution along stirrups.

Reference was made previously to the effect of the finer mesh of cracks and to the difference in the behaviour of the inner and outer stirrups. The comments made are supplemented with Fig. 6.95 in which the strain distribution along each of the instrumented stirrups is shown at the maximum load level of the first four load cycles. The different strain patterns for the positive and negative load cycles in the outer stirrups are particularly evident. The curves may also be compared with those obtained by Beam 313 in Fig. 6.76.

6.4.5 Deformations

6.4.5.1 Rotations.

The load-rotation relationship for each end of Beam 314, as shown in Fig. 6.96 confirms that only small plastic rotations have occurred at the end of the loading cycles. For this reason the "soft range" at low loads is also considerably smaller than in the previous tests of this series.

The beam behaved quite symmetrically. In the first load cycle no difference between the two end-block rotations could be detected. Only at the end of the 4th cycle could slightly larger plastic rotations be observed at the right hand support.

6.4.5.2 The variation of stiffness.

The interpretation of the load-rotation curves in terms of stiffness is presented in Fig. 6.97. The great loss of stiffness at the beginning of the first cycle is of the same order as that observed for Beam 313. The first load increment produced already flexural cracks, so there was no clear evidence how close the actual stiffness of the uncracked beam was to the theoretical value.

A slight increase of the maximum stiffness can be observed in the 3rd and 4th cycles, i.e. when cracks in both directions have already fully developed. It is difficult to assess whether this was due to the presence of the # 5 secondary reinforcement or due to the smaller plastic deformations which the beam was subjected to in the previous cycles.

6.4.5.3 The elongation of the beam.

The curves obtained from the displacements of the end blocks, and reproduced in Fig. 6.98, are identical in form to similar curves obtained for the other beams of this series.

The loading branches of the curves are somewhat steeper indicating a larger stiffness with respect to elongation. The # 5 secondary bars, which restrain elongation, are responsible for this.

The only notable difference is the absence of large plastic elongations. This was to be expected as no large yielding of the flexural reinforcement was observed.

6.4.5.4 Transverse expansion.

The transverse (vertical) expansion was measured in exactly the same way as in the previous beam. The curves, presented in Fig. 6.99 indicate near perfect symmetry in behaviour. In the last (5th) load cycle the approaching distress near the supports is evident. In the first four cycles the maximum deformations have increased by a small amount only indicating a small degree of deterioration. However, the sudden increase of transverse expansion across the beam adjacent to the supports indicates that after two cycles of loading in each direction a "shear slip" occurred across the compression zones of the beam. It was pointed out earlier that this phenomenon engages only the stirrups in the immediate vicinity of the supports.

The failure, as revealed by Fig. 6.100 and Fig. 6.102, is consistent with a "shear slip" mechanism.

6.4.5.5 Deflections.

The deflections, which were measured in the usual manner, have revealed no significant new features. For this reason the results are not reproduced here.

Very large shear displacements occurred at failure across the failure section. They are clearly visible on Fig. 6.101 which shows the affected area after the loosened cover over the reinforcement has been removed.

6.5 A Comparison of Medium Coupling Beams

Only those features of the behaviour of Beams 311, 312, 313 and 314 form the basis of this comparison which were likely to be influenced by the major variable in the test, the web steel content. An analytical approach is also presented to predict the distribution of tensile forces along the flexural reinforcement.

6.5.1 The Behaviour of the Flexural Reinforcement

6.5.1.1 Strain distribution along the beams.

Fig. 6.103 and Fig. 6.104 show the mean strain distribution along the top and the bottom reinforcement for all four beams of this series at two load levels. The effect of different stirrup contents is not very pronounced. However it is noticeable that steel strains are generally smaller, particularly at the higher load, when the amount of web reinforcement is increased. Beam 312 is an exception. For unknown reasons this beam exhibited the largest strains at low loads and also at higher loads over the "tension zone" of the beam. The intermediate horizontal bars in the web of Beam 314 account for the particularly low strains.

The increased stirrup content considerably reduced the strain difference between the two layers of the flexural reinforcement in the "compression zone" of the beam. A comparison of Fig. 6.5, Fig. 6.29 and Fig. 6.61 reveals this clearly. The phenomenon is associated with a diagonal tension failure mechanism, which occurred in Beam 311 only.

6.5.1.2 An analytical study of the tension force distribution.

An evaluation for the observed distribution of tension forces along a coupling beam may be obtained by considering the internal equilibrium conditions after cracking. Certain idealisations, particularly in the formulation of a mathematical model, are inevitable in such a study.

Numerous photographs have shown a similar crack pattern for all beams of this series. A simplified diagram, Fig.6.105.a shows these idealised cracks radiating from the compression corners of the beam. A section taken along the line S-S isolates a part of the beam, which may be further examined. The internal forces along this cut, as shown in Fig. 6.105.b, may then be identified as follows:

a.) The tensile force in the top reinforcement is T, the intensity of which this study aims to approximate.

b.) The compression force, C, is the sum of the horizontal concrete and steel forces in this area.

c.) Aggregate interlock forces along the crack and the appropriate component of the diagonal concrete compression zone of the beam, g.

d.) The dowel force, V_{do}, induced across the bottom reinforcement at the left hand support.

e.) The dowel force, V_d, induced across the top reinforcement at distance x from the left hand support.

f.) A vertical distributed load, p_s, along the diagonal, represents the closely spaced stirrup forces. The intensity of p_s is assumed to be uniform over the length of the beam in spite of the fact that this violates the requirements of strain compatibility within the cracked beam. The problem is examined more closely in Chapter 9. These forces may be conveniently expressed in terms of the total shear force, V_s, which is resisted by all the stirrups crossing the main diagonal crack, thus

$$p_s = \frac{V_s}{l}$$

This force as assessed from measurements for each test beam during the first cycle of the loading.

g.) The tension force T' represents the resultant of the forces generated in the secondary horizontal web reinforcement. The effect of this force is examined in Chapter 7, where the behaviour of the horizontal web reinforcement is presented in greater detail. For the first three beams of this series $T' = 0$.

h.) The external load is P and the moment generated at the support is $M = Pl/2$.

By considering the moments about the lower left hand corner of the beam (Fig. 6.105.b) it is found that:

$$M = zT + xV_d + x^2\frac{p_s}{2} \qquad (6.1)$$

The dowel force, V_d, is relatively small. In studies of shear problems it is usually neglected. An estimation of its maximum contribution can be made however if it is assumed that the reinforcement has to carry the dowel shear on a two inch (half stirrup spacing) lever arm. In such a case 2 # 7 and 2 # 8 bars could carry approximately 7.6 Kips shear when flexural yielding sets in. In fact a considerably smaller load would be transfered across the bars because they are already stressed in axial tension and thus could not receive appreciable additional flexural stresses. It will be assumed that the intensity of the dowel force in the top reinforcement varies from zero at the left hand support, where the flexural steel is subject to maximum tension, to the maximum value, V_{do}, at the right hand support. The maximum dowel capacity, is approximately 5% of the ultimate load carried by these beams and, therefore, it could be approximated in this form

$$V_d = \frac{x\,V_{do}}{lP_u}P = 0.05\xi P \qquad (6.2)$$

When p_s is expressed in terms of f.) and $x/l = \xi$ is used, the tension force intensity along the beam is found from Eq. (6.1)

$$T = T_m\left[\,1 - (.10 + \eta)\,\xi^2\right] \qquad (6.3)$$

where $T_m = \frac{pl}{2z}$ is the maximum tension generated at the vertical section of the left hand support.

The term ".10" represents the maximum contribution of the dowel force in these test beams. The effectiveness of the web reinforcement is measured by the $\eta = V_s/P$ ratio.

The meaning of Eq. (6.3) is illustrated by Fig. 6.106, where the tension force intensity along a coupling beam is expressed in terms of the maximum tension, T_m , as a function of the web reinforcement's contribution. The contribution of dowel forces has been neglected in these curves.

To illustrate the relevance of these curves one may consider a beam with an aspect ratio of $l/D = 1.5$, in which the stirrups, designed in a conventional manner, are expected to accept 80% of the shear force P . Across the critical main diagonal these stirrups would resist, at their given capacity, a force $V_s = 1.5 \times 0.8P = 1.2P$, so that $\eta = 1.2$. The appropriate curve in Fig. 6.106 indicates that the flexural reinforcement would be in tension over approximately 90% of the span.

When the same stirrups are used in a square beam (1/D = 1), so that $V_s/P = .8$, then the tension force at the "compression end" of the beam should be approximately 20% of the value attained simultaneously at the tension end.

After diagonal cracking theoretically the tension should be uniform over the whole length of the beam if no web reinforcement is provided. This implies that the shear force must be resisted by a linear arch.

The theoretical upper limit is represented by the case when the whole shear force is resisted by stirrups. In a beam with an aspect ratio of 2 the tension would extend over 70% of the span and in a square beam over the entire span.

This study shows that no matter how much stirrup reinforcement is provided, the flexural steel will always be subjected to tension in the "compression zone" of a diagonally cracked coupling beam.

When Beam 311 carried a load of 118 Kips the stirrups, according to Fig. 6.16 received 74% of this load. Hence with this information and the usual approximation for the internal lever arm (z 25 in.) the tension force is obtained from Eq. (6.3) thus:

$$T = 94.5(1 - .84\,\xi^2)$$

The experimental values for the top and bottom reinforcement agree well with this expression as may be seen in the top diagram of Fig. 6.107. The shaded area between the two theoretical curves indicates the insignificant influence of the dowel action. Similar comparisons are made in the other two diagrams of Fig. 6.107 for Beam 312 and 313. Considering the approximations, which were made, particularly with regard to the uniformity of stirrup forces and the magnitude of the internal lever arm, the agreement in all cases is satisfactory. If allowance would have been made for the reduced internal lever arm, as shown in Fig. 6.9 and Fig. 6.32 an even better agreement would have been obtained.

6.5.1.3 Bond stresses.

An examination of the tension force distribution indicates certain new features with respect to the mean flexural bond stresses.

By differentiating Eq. 6.3 the bond force per unit length of beam is determined thus

$$\frac{dT}{dx} = \frac{2T_m}{l}(.10 + \eta)\ \xi \qquad (6.4)$$

This may be compared with the (conventional) bond force that would occur if the rate of change of the internal forces would correspond with that of the bending moments. It is found that

$$\frac{\text{Bond Force in Coupling Beam}}{\text{Conventional Bond Force}} = (.10 + \eta)\ \xi \qquad (6.5)$$

From this equation and the re-examination of the diagrams shown in Fig. 6.107, it may be concluded that the bond force;

a.) varies along the beam in spite of the constant shearing force;

b.) is zero (within the approximations of the idealised model) at the section of maximum tension;

c.) is proportional to the maximum fraction of the shear (η) resisted by the stirrups;

d.) is smaller than the value indicated by the conventional analysis, everywhere along the span, except perhaps at the "compression end" of the beam;

e.) may theoretically attain, in the critical compression zone, the value predicted by the customary bond equation, when the stirrups resist approximately the whole of the external load. However, the zone of these high bond stresses is seriously disturbed, particularly at high loads, by local discontinuities (Fig. 6.19.c) in the cracked beam. This disturbance is likely to be responsible for the radical loss of bond in the critical compression zone. The tension curves for Beam 312 and 313, shown in Fig. 6.107, demonstrate the phenomenon clearly.

The intensities of the theoretical (conventional) bond stresses in these beams remain within the limits recommended by the AC I Code.

6.5.2 The Behaviour of Stirrups

Measurements indicated that stirrups which are situated at the centre portion of the beams are always more highly stressed during the first load cycle. A comparison of the load-stress relationships with the AC I recommendations reveals that

a.) The diagonal cracking load, which is rather inconsistent, is satisfactorily predicted by the code equation.

b.) The mean stress in the stirrups agrees well with the value obtained from the customary truss analogy. This is demonstrated in Fig. 6.108, which presents the mean stresses, based on the four largest adjacent strain readings, for the most highly stressed central (No.7) stirrup of each beam.

c.) Generally the most highly stressed point along a stirrup lies where the major diagonal crack is being crossed. The stresses at these points usually exceed the theoretical values. The load-stress relationship at the critical point of the central stirrups is presented in Fig. 6.109.

It was shown that the strength of the web reinforcement could be conveniently assessed by evaluating the strength of the stirrups across the main diagonal. This information gave a good indication as to the type of failure to be expected. Fig. 6.73 and Fig. 6.93 show that the strength of the web reinforcement across this critical crack exceeded the applied load. Beam 312 (see Fig. 6.43) represents a border line case. If the load would have been increased to failure during the first load cycle, most probably a diagonal tension failure would have occurred. The strength of the web reinforcement in Beam 311 (Fig. 6.16) was clearly inadequate.

The distribution of strains along stirrups is not uniform. A definite strain pattern, consistent with the shear failure mechanisms of the beams is apparent. The variation of stirrup stresses indicate that the surrounding concrete of the web absorbs bond forces. These became relatively small when the size of the stirrups is increased. Consequently the strain distribution along larger diameter stirrups was found to be more uniform.

Under high intensity alternating loading the contribution of the concrete towards shear strength diminishes, even if the stirrups perform entirely in the elastic range. (Fig. 6.94).

Beam 312, which contained insufficient web reinforcement displayed considerable ductility in shear. (Fig. 6.46, Fig. 6.47, and Fig. 6.48).

The role of outermost stirrups became evident during alternating near-ultimate loading. Because of the deterioration of the compression zone, during load reversals, shear slips occurred. A few stirrups in the immediate vicinity of the supports attempt to counteract the separation of the beam along the steep diagonal cracks. For the beam to fail these stirrups must always yield. (See Fig. 6.75).

A comparison of the stirrup forces, generated at different stages of the loading revealed no special features. Contrary to the finding of Mayer[64] there was no clear evidence that heavier stirrups have taken a larger share of the load. The mean stirrup forces generated in the three instrumented central stirrups during the first load cycle are compared for all four beams of this series in Fig. 6.110.

6.5.3 Deformations

6.5.3.1 Rotations.

To assess the influence of the web reinforcement upon the stiffness, the load-rotation curves for all four beams of this test series are assembled in Fig. 6.111.a. The first diagram shows the comparison during the first loading cycle. Unfortunately the concrete strength of the specimens differed more than it was desired. This probably had little effect in the uncracked state, as indicated by the straight dotted lines. However, the elastic properties of the concrete may have influenced rotations more significantly in the cracked state. An arbitrary parameter, $p_w\sqrt{f'_c}$, which allows for the principal variables in these beams, is also recorded in the diagram.

There is a considerable difference between Beams 311 and 312. The former failed in diagonal tension and the latter in diagonal compression and sliding. The smaller stiffness of Beam 311 is due to the small web steel content and the early yielding of the stirrups. The stiffening effect of the intermediate bars used in the web of Beam 314 is apparent.

A comparison during the 2nd and 3rd load cycles, shown also in Fig. 6.111.a, reveals a similar pattern for all beams. The influence of the web reinforcement can not be identified easily because of the effect of different plastic rotations imposed at the end of a previous load cycle. The secondary reinforcement used only in Beam 314 restrained the yield deformations during cyclic loading, and was responsible for the greater stiffness of that beam at all stages.

Beams 312 and 314 are compared during the 4th to 6th cycles in Fig. 6.111.b.

In Chapter 9 a theoretical approach is presented for the approximate determination of the stiffness of coupling beams in the cracked state. The theory is then compared with the experimental results for all beams.

6.5.3.2 Beam elongations.

A comparison of the beams, which contain identical horizontal reinforcement, was made in Fig. 6.55. Very little difference is noticeable in the elongation characteristics. It is to be noted that the measured elongation also includes extensional deformations and slips in the anchorage zones.

The elongation of the beam proper can be estimated with sufficient accuracy by the summation of the steel strains in the top or bottom reinforcement.

$$\text{i.e.} \quad \Delta'_H = \frac{1}{A_s E_s} \int_0^l T(x)\, dx \qquad (6.6)$$

where $T(x)$ was defined by Eq. (6.3). By completing the integration it is found that

$$\Delta'_H = \frac{T_m l}{A_s E_s} \left[1 - \left(\frac{.10 + \eta}{3} \right) \right] = C_H \frac{T_m l}{A_s E_s} \qquad (6.7)$$

when dowel action is considered. The extreme limits of the elongation of the beam proper are as follows

$$1 \geqq C_H \geqq .633$$

when $0 \leqq \eta \leqq 1$

and $0 \leqq \frac{V_{do}}{P_u} \leqq .05$

An approximate analysis indicated that at the end of the first cycle only about 25% of the total (.09 in.) elongation was caused by elastic extension of the beam proper. (See Fig. 6.55). The remainder resulted, in approximately equal proportions, from elastic deformations in the anchorages, slip, and concentrated yield at the critical sections. With cyclic loading the relative proportions of the component elongations change because with alternating load the elastic and plastic strains increase in the beam proper.

The stiffening effect of intermediate bars, used in Beam 314, are demonstrated in Fig. 6.98. The curve showing the elongations in the first load cycle indicates the magnitude of the slip in Beam 314. This is the major part of the permanent elongation after removal of the elastic load. It appears that no further slip occurred after the 2nd load cycle.

6.5.3.3 Transverse expansions.

The maximum transverse expansions, measured at midspan, are compared for the 1st and 2nd load cycles in Fig. 6.112. The effect of increased stirrup reinforcement is very marked. The additional horizontal web reinforcement, used in Beam 314, had no apparent influence upon the performance of the stirrups.

CHAPTER SEVEN

DEEP COUPLING BEAMS

The test series consisted of four beams numbered 391 to 394. The properties of these beams were summarised in Chapter 5. The overall dimensions of the four beams were the same. The span to depth ratio was

$$\boxed{\frac{l}{D} = 1.02}$$

The web reinforcement and the load sequence have been varied.

7.1 Beam 391

7.1.1 Loading and Test Procedure

The beam was loaded in one way only in 13 increments till failure occurred. The aim of the test was the same as that for Beam 311 described in Chapter 6.

The load sequence and other relevant data are presented in Table 7.I.

A view of the reinforcement in the beam proper, also showing the gauge points for strain measurements, may be seen in Fig. 7.1. The crack pattern of the beam after failure along the main diagonal is shown in Fig. 7.2. The gauges, which enabled the diagonal concrete compression strains to be determined after the cracks were well developed, can also be located on this photograph.

7.1.2 The Behaviour of the Flexural Reinforcement

7.1.2.1 The distribution of strains.

Steel strain measurements followed the same procedure as that used for the previous test series described in Chapter 6. The mean strains for two # 7 bars are plotted separately for the outer layer (solid points) and inner layer (small circles) of reinforcement in Fig. 7.3 for four load increments.

The general pattern is similar to that observed in the previous beams. The deviation from the moment pattern is already pronounced when the load was 50% of the theoretical ultimate. At this stage of the loading already near uniform tensile strains were generated over half the length of the beam proper. At 84% of the theoretical ultimate load yielding extended over approximately 12 in. length of the reinforcement.

The deviation of the strains between the two layers of # 7 bars in the "compression zone" is very pronounced as the load increases. The phenomenon was examined in 6.1.5 and illustrated in Fig. 6.19.c.

7.1.2.2 The tension force distribution.

From the mean strains the tension force, represented by the four # 7 bars in each face of the beam, was determined and plotted in Fig. 7.4. The theoretical load-tension force relationship, shown by the straight dotted lines, was based on the conventional elastic approach taking into account the intermediate # 3 bars. The position of the horizontal bars and the load pattern are also indicated at the top of Fig. 7.4.

7.1.3 <u>The Performance of the Horizontal Web Reinforcement</u>

Intermediate horizontal bars were first used in this project in Beam 314. A discussion related to their expected and observed behaviour was presented in 6.4.3, and the test results were shown in Fig. 6.90. The strain distribution along each of the three layers (see upper diagram of Fig. 7.4) of # 3 bars of Beam 391, is shown in Fig. 7.5.

The theoretical strains, based upon conventional analysis, are indicated by the dotted lines.

After the full development of the cracks a more or less uniform strain distribution was observed along the beam with one exception. At high load intensities the strains increased considerably where the intermediate horizontal bars were crossed by the main diagonal crack. This is understandable for this crack is associated with the failure mechanism of the beam. The web reinforcement was deliberately made insufficient so as to ensure failure of the beam by separation into two halves along the main diagonal. At low loads the performance of these, relatively small size, bars was greatly affected by the formation of cracks. The intermediate bars seemed to have encouraged the early formation of the main diagonal crack.

7.1.4 <u>The Behaviour of the Stirrups (# 3)</u>

7.1.4.1 Load-stress relationship.

The load-stress curves at the four most highly stressed locations of four stirrups are reproduced in Fig. 7.6. The locations of the strain measurements are indicated in a key diagram at the top of the page. It is evident that the largest strains along the stirrups occurred where the same were crossed by the main diagonal crack. This was to be expected because the web reinforcement was assessed as being able to resist only 55% of the theoretical ultimate flexural load. The load-strain curves at the points of intersection of the stirrups and the main diagonal line are shown by heavy lines in Fig. 7.6. The behaviour of these stirrups was very similar to those provided in Beam 311. (See Fig. 6.12).

The diagonal cracking load is estimated from these curves to be 77^{K}. (v_{de} = 355 p.s.i.)

7.1.4.2 The strain distribution along stirrups.

The previously found pattern of strain distribution was confirmed also in this beam. The strain curves shown for four load increments in Fig. 7.7 clearly indicate the critical nature of the main diagonal crack. It is interesting to note that when the load was increased from 50% to 75% of the theoretical ultimate, only small strain increases occurred in the regions of stirrups which were situated sufficiently far from the major diagonal crack.

7.1.4.3 The ultimate strength of the web reinforcement.

Because of the aspect ratio of the beam the main diagonal crack happens to be at 45°. This enables the conventional truss analogy (AC I) to be directly compared with the experimental results.

The contribution of the stirrups, which crossed the main diagonal, was assessed from the measurements at each load level. The procedure used was the same by which the contribution of the stirrups was determined in the previous series of test beams. (See Fig. 6.15). The shear resisted by the stirrups across the failure crack is shown against the load in Fig. 7.8, from which it is evident that the concrete mechanism carried a considerable portion of the shear at all stages. For the sake of comparison the AC I recommendation is also indicated in this diagram.

7.1.5 Diagonal Concrete Compression Strains

At the end of the first day of testing, when 50% of the theoretical ultimate load was applied to the beam, the diagonal cracks were well developed. At this stage gauge points, for 2 in. Demec Gauges, were attached to the concrete between cracks near the supports and at the centre of the beam. They were oriented so as to be approximately parallel with the axes of the diagonal concrete struts formed between cracks. The previous load increments were repeated so that the strains could be determined in the cracked beam.

The load strain relationship and the locations of the gauges are shown in Fig. 7.9. Up till 60% of the ultimate load a satisfactory agreement was found with the theoretical stress-strain relationship at the centre of the beam. Near the supports the diagonal strains in the flexural compression zone increased rapidly as the ultimate load was being approached.

The curves may be compared with those drawn for Beam 312 in Fig. 6.52.

7.1.6 The Failure Mechanism

The development of the failure mechanism in this beam was essentially the same as for Beam 311 described in 6.1.5.

7.1.7 Deformations

The deformation characteristics of coupling beams and the means by which these were evaluated were discussed in Chapter 6. For this reason only special features, if any, are reported here.

7.1.7.1 Rotations.

The rotation measurements carried out at the end-blocks indicated a near perfect antisymmetrical behaviour up till 80% of the theoretical ultimate load. The load-rotation relationship, with distinct stages of the behaviour, is shown in Fig. 7.10.

7.1.7.2 The elongation of the Beam.

The relative movements of the end blocks, ΔH, were very similar in this beam to those observed in Beam 311 (see Fig. 6.22). The load-ΔH relationship is also shown in Fig. 7.10.

7.1.7.3 Transverse expansion.

The symmetrical behaviour of the beam, at least up till 75% of the theoretical ultimate load, is revealed by its transverse expansion, as seen in Fig. 7.11. Owing to larger shear forces and longer stirrup length the transverse expansions are larger than those observed in Beam 311, the counterpart of this beam. (See Fig. 6.24). The non-linearity of the load-maximum transverse relationship immediately after diagonal cracking is also evident in this beam. (see also Fig. 6.25).

7.1.7.4 Deflections.

Fig. 7.12 shows the deflected shape of the beam at three load levels. The significance of the transverse expansion in determining the distorted axis of the beam is apparent. The predominance of shear distortions is evident. A fuller explanation of the phenomenon is given in Chapter 9 where distortions are examined analytically.

7.1.8 Crack Widths

The development of the failure mechanism could further be verified by measuring the width of the cracks. The results of these measurements are presented in Fig. 7.13. The widening of the cracks along the main diagonal was considerably greater than in other areas of the beam. The width of the main diagonal crack is largest at midspan as can also be expected from the transverse expansion measurements shown in Fig. 7.11.

7.2 Beam 392

7.2.1 Loading and Test Procedure

The beam proper was in every respect identical to Beam 391 except for the slightly higher strength of the concrete. The aim of this test was to study the beam's behaviour under cyclic loading.

Time consuming strain measurements on the reinforcement were made only during five load cycles, but the rotation of the end-blocks was observed throughout the 13 cycles of loading so as to be able to assess the changes in beam stiffness. The behaviour of the intermediate horizontal # 3 bars was not examined in this test. Four days were required for the test.

Relevant data on the load sequence are summarised in Table 7.II.

7.2.2 <u>The Behaviour of the Flexural Reinforcement</u>

7.2.2.1 The distribution of strains.

For both layers of # 7 bars in the top and bottom of the beam the strain distributions are presented, for the 1st load cycle only, in Fig. 7.14. These curves are very similar to those obtained for Beam 391. (See Fig. 7.3). The maximum load was deliberately kept low so that no excessive yield should occur in the stirrups during the first cycles. Consequently the strains for the flexural reinforcement are well within the elastic limit. The width of the main diagonal crack, which was one of the first to develop, was still small during the first load cycle and so the strain differences in the steel layers of the compression zone (at gauge locations 19, 20 and 41, 42) were not excessive.

7.2.2.2 The tension force distribution.

Because the behaviour of the reinforcement in the top and the bottom of the beam was very similar, the tension force distribution is shown in Fig. 7.15 for the 4 top bars only. The full lines indicate the situation in the first two load cycles and the dotted lines refer to the 9th and 10th cycles respectively.

It is evident that steel stresses are higher throughout the beam after cyclic loading. The increase is particularly large at low load increments.

At the first gauge points, adjacent to the supports, a kink appears in the curves after the first load reversal. This is due to the serious local disturbance caused by the main diagonal crack, where this enters the compression zone of the beam. Because of the large strain differences between the two layers of the reinforcement in this area, the mean strain can not be evaluated accurately.

The curves demonstrate again that after the first load cycle the reinforcement carries tensile forces over the entire span of the beam.

The maximum tensile force near the support is predicted by the conventional analysis with reasonable accuracy in the first cycle of loading only. (Up to .65 P_u^*). In subsequent load cycles the critical tensile force generated is invariably larger than the theoretical value.

It is to be noted that up till the application of the 9th cycle no yield was caused in any part of the flexural reinforcement.

7.2.3 <u>The Behaviour of the Stirrups (# 3)</u>

7.2.3.1 The strain distribution along the stirrups.

As expected the performance of the stirrups in Beams 391 and 392 was very similar during the first load cycle. This is particularly evident when the strain distribution curves for this beam, given in the top half of Fig. 7.16, are compared with the corresponding curves of Beam 391, shown in Fig. 7.7. The sharp rise of strain at the points where the main diagonal crack crosses the stirrup is also apparent. The diagonal cracking load was unusually low at approximately 47^K (v_{dc} = 217 p.s.i.)

The redistribution of strains during the second loading cycle, shown in the bottom half of Fig. 7.16, corresponds with the new, antisymmetrical load pattern. The effect of the main diagonal crack is particularly noticeable at the first (No.5) and last (No.9) instrumented stirrups, adjacent to the supports of the beam. These curves show that little load is carried by these two stirrups in the tension zone of the beam. This behaviour is consistent with the pattern of tension force distribution of the flexural reinforcement, shown in Fig. 7.14 and Fig. 7.15, which indicate the absence of significant bond forces in this region.

It is to be noted that in the second load cycle already residual strains, particularly in stirrups 7 and 8, are present at the critical gauge locations.

The strains generated during the 9th cycle, and presented in Fig. 7.17, can be directly compared with the first cycle strains. (Fig. 7.16). The load was applied in both cases in a positive sense. This diagram very clearly shows the critical regions for each stirrup. (The position of the gauges relative to the main diagonal crack can be seen in Fig. 7.6.) Because of the higher load applied at the end of this cycle the yield strains have increased very considerably.

The effect of alternating loading is evident when the strain distribution curves for Beams 391 and 392 are compared for the same, relatively high intensity, loading, i.e. Fig. 7.7 and Fig. 7.17 at $P_i/P_u^* = .75$ to $.76$. The yield strains are considerably larger in Beam 392.

The strains observed during the 10th load cycle, recorded in Fig. 7.18, are markedly affected by the permanent strains induced in the stirrups during the previous cycle of high load intensity. The new large yield strains along the main diagonal crack, which corresponds with the negative loading, can be recognised. It is to be expected that these are associated with large transverse expansions of the beam.

7.2.3.2 The strain history during cyclic loading.

No new features of the strain history of stirrups in this beam were observed. Therefore only two typical cases are presented in Fig. 7.19. Gauge point 82 is primarily affected by positive and gauge point 85 by negative loading. (See the crack pattern of the beam in Fig. 7.20.c.). The yield deformations are imposed upon the stirrups accordingly.

During the 10th cycle the closure of the main diagonal crack first imposed compression strains upon that position of the stirrup which had extensively yielded during the previous cycle. (Gauge points 82 and 75 in Fig. 7.19).

The central stirrups (Nos. 7 and 8) yielded at the critical gauges, during the first cycle, at a load less than that predicted by the A C I Code.

7.2.3.3 The ultimate strength of the stirrups.

The share of the stirrups in carrying the external shear across the main diagonal crack was assessed in the same manner as described for previous beams. To enable a comparison to be made the results are presented together with those for Beam 391 in Fig. 7.8. The contribution of the stirrups is very similar in both beams except at low loads. Diagonal cracking set in considerably earlier in Beam 392.

The diagonal cracking load is estimated at 47K. (v_{dc} = 217 p.s.i.).

7.2.4 The Failure Mechanism.

Beams 311, 391 and 392 were all deliberately under-reinforced in shear. Hence the failure mechanism is associated with the main diagonal crack.

Fig. 7.20.a. shows the beam when it carried 30% of the theoretical ultimate load and when the main diagonal crack was formed. The same crack appeared soon after the reversal

of the load at $P_i = -.30 P_u^*$, as can be seen in Fig. 7.20.b. The photograph also shows the regular crack pattern which developed at the end of the first cycle. Failure occurred by separation along the main diagonal at the end of the 13th cycle at 80% of the theoretical ultimate load. The beam at this stage is shown in Fig. 7.22.c. After the removal of the load the failure crack was approximately ¼ in. wide. The beam carried 95% of the failure load of its companion, Beam 391.

Without taking any measurements the load was once more reversed. This resulted in the closure of the previous large diagonal failure crack. Surprisingly the beam failed at a high load of 74% theoretical ultimate or 93% of the previous failure load by disintegrating. The specimen, as it appeared in Fig. 7.20.d. was very similar to coupling beams that were destroyed by earthquakes. (Also see Fig. 1.4).

7.2.5 Deformations

7.2.5.1 Rotations.

For the sake of clarity the load-rotation curves are presented in three groups, for the 13 cycles of loading, in Fig. 7.21.

The curves representing the behaviour of Beam 392 in the 1st and 2nd load cycles show the familiar pattern associated with cracking. Up till the 9th load cycle the elastic behaviour of the beam is evident. Once the cracks have fully developed there is no significant difference between the load rotation curves, i.e. 3rd to 9th cycles. The behaviour of the beam was antisymmetrical to such a degree that differences of end-rotations could not be suitably shown to the scale used in Fig. 7.21. The curves thus represent the rotation of both ends.

The yield deformations imposed at the end of the 9th, 10th and 13th cycles are rather small. Their effect upon the softening of the beam at very small loads however is more significant.

To enable an improved examination of the beam to be carried out at the onset of the loading, i.e. in the "soft range", a number of small increments were used at the beginning of each cycle.

7.2.5.2 The variation of stiffness.

The variation of beam stiffness with cyclic loading is represented by Fig. 7.22. The characteristics of these curves have been discussed in Chapter 6. The "hardening", "steady" and "softening" ranges after cyclic loading are also evident in this beam.

During the elastic cycles, after the full development of cracks, the measured stiffness of the beam was approximately 26% of the value obtained for the uncracked beam.

After some plastic excursions the "steady range" of the measured beam stiffness reduced further to approximately 18% of that of the uncracked beam.

7.2.5.3 The elongation of the beam.

The beam elongation determined during the test is presented in Fig. 7.23. It is evident from these curves that a permanent elongation of about .015 in. occurred after the first load cycle, in spite of the fact that no yielding was induced in the flexural reinforcement. This set was most probably due to slip at the anchorages and residual tensile strains in the reinforcement at no load, i.e. inability of the cracks to close after load removal.

The elastic behaviour of the beam is well demonstrated by the closely spaced curves representing the subsequent cycles. The permanent set owing to yielding of the flexural reinforcement in the 9th and 10th cycles, can be recognised.

It appears that slip at the anchorages did not increase during cyclic loading.

7.2.5.4. Transverse expansion.

The gradual increase of plastic deformation with cyclic loading had a marked effect upon the transverse expansion of the beam. These are shown for five significant cycles of the loading in Fig. 7.24.

The deterioration of the beam is demonstrated by the transverse expansion - load relationship at midspan, as shown by Fig. 7.25. For the sake of comparison the results of the companion beam (391) are also superimposed.

7.3 Beam 393.

7.3.1 Loading and Test Procedure

In order to avoid a diagonal tension failure, the size of the stirrups was increased. For the vertical and horizontal web reinforcement # 4 bars were used.

Strain measurements on all reinforcement were made during the 1st, 2nd, 7th, 8th and 11th cycles of loading. The rotations of the beam were recorded throughout the test.

The load sequence for the beam is given in Table 7.III.

7.3.2 The Behaviour of the Flexural Reinforcement

7.3.2.1 The distribution of strains.

The strains induced in all layers of the flexural steel during the first load cycle are shown in Fig. 7.26, for three load increments. The curves confirm the previously identified behaviour. Yield has not occurred.

7.3.2.2 The tension force distribution.

Only for the top reinforcement are the tension forces presented in Fig. 7.27. The behaviour of the bottom steel was similar. Because of the large permanent strains produced during the 7th cycle the steel forces near the left hand support could not be determined with a satisfactory degree of accuracy during the 8th load cycle. The gradual increase of stresses with repeated loading is again evident.

The critical strains at the supports have been examined and the mean values, derived from both ends of the beam, are plotted against the load for three load cycles in Fig. 7.28. The curves give qualitatively the increase of stresses with cyclic loading.

7.3.2.3 Load - Strain history.

The effects of cyclic loading and the effect of maximum load intensity within one cycle are demonstrated by the load-strain curves of the top reinforcement as shown in Fig. 7.29.

At the left hand end of the beam the top reinforcement, at gauge points 1 and 2, was situated in the tension zone during positive (odd numbered) load cycles. The behaviour here was elastic throughout the first seven cycles. After the 7th load cycle a permanent strain of approximately 15500 microstrains, occurred. This large strain necessitated compression yielding at least in one layer during the subsequent reversed load cycle.

Extensive yielding occurred at the right hand end of the beam at the end of the 8th cycle. This steel is situated in the tension zone, at gauge points 21 and 22, during the negative (even numbered) load cycles.

7.3.3 <u>The Performance of the Horizontal Web Reinforcement</u>

The strains along the #4 intermediate bars are presented in Fig. 7.30 for the top layer, in Fig. 7.31 for the central layer and in Fig. 7.32 for the bottom layer. (The position of the layers is shown in a key diagram of Fig. 7.4). In each figure the top diagram records the performance during the positive (1st and 7th) and the bottom diagram during the negative (2nd and 8th) load cycles.

The measured strains bear little relation to the theoretical values. An approximately uniform strain is generated over the full length of these bars. The average strain is approximately the same as induced in the flexural reinforcement at midspan.

It is interesting to note that at the last increments of the 7th and 8th load cycles yielding occurred in all intermediate bars. For example, the layer at middepth of the beam has extensively yielded at the centre of the beam. (Fig. 7.31). This was brought about by the excessive elongation of the beam caused by the large yielding of the flexural reinforcement at the supports.

7.3.4 The Behaviour of the Stirrups (#4)

7.3.4.1 The load-stress relation in the first cycle.

A satisfactory agreement with the ACI recommendation is demonstrated by the load-strain curves for stirrups shown in Fig. 7.33. The position of the stirrup, where it is crossed by the major diagonal crack in the first cycle does not show special features. (The stresses at these gauge locations are shown by heavy lines). This was also observed in Beam 313 where adequate web reinforcement prevented a diagonal tension failure along the main diagonal.

7.3.4.2 The strain distribution along the stirrups.

During the first two cycles of loading all stirrups performed elastically. Fig. 7.34 shows that the strain distribution during these cycles is rather uniform with the exception of the outer stirrups. Their ineffectiveness in the tension zone of the beam is again demonstrated.

After six cycles of alternating elastic loading the concrete surrounding the stirrups does not appear to have accepted appreciable load by means of bond. Consequently near-uniform strains prevailed over the entire height of the stirrups. The curves drawn for the 7th load cycle in Fig. 7.34 illustrate this well. The outer stirrups are exceptions. At the last load increment of the 7th cycle all stirrups yielded at points which lie on the main diagonal. If the load would have been further increased at this stage of the loading, the beam probably would have failed in diagonal tension.

The strains induced during the 8th and 11th load cycles are presented in Fig. 7.35. The large permanent strains imposed upon the stirrups at the end of the 7th cycle and new yield strains at the highest load level, characterise the strain curves of the 8th cycle. It is particularly interesting to note that heavy yielding occurred in the outer stirrups where the same entered the compression zone of the beam. This suggested that considerable shear displacements must have occurred in the damaged compression zones.

During the 11th cycle relatively low intensity of loading was applied so that all stirrups behaved elastically. The large variation of strains along each stirrup is entirely due to the elastic deformations imposed upon the web reinforcement during the 7th and 8th cycles.

7.3.4.3 The ultimate strength of the stirrups.

The contribution of the stirrups in carrying the external load across the main diagonal is shown for the 1st and 7th load cycles in Fig. 7.36. At high loads about 76% of the shear was carried by the web reinforcement and this agrees well with the ACI recommendations.

The diagonal cracking load is approximately 70 K (v_{dc} = 322 p.s.i.).

7.3.5 The Failure Mechanism

The beam was under-reinforced for shear and it would have failed along the main diagonal if the load in the first cycle had been increased sufficiently. The repeated alternating loading, particularly the 7th and 8th load cycles, damaged the compression zones sufficiently so that the cause of failure appears to be diagonal crushing and sliding shear across the right hand support. The crack pattern and strain measurements indicated that very large yield strains occurred in the stirrups, such as No.9, entering the critical compression zone. The cause of the failure is primarily the excessive yielding of the stirrups at the supports. Fig. 7.37 shows the beam at a stage when the loading was half way through the 2nd cycle. New cracks were formed at right angles through the fully developed cracks of the 1st load cycle. (Exceptionally the beam is viewed from the West). The beam after failure may be seen in Fig. 7.38.

7.3.6 Deformations

7.3.6.1 Rotations.

For the first 8 cycles of loading the rotation curves are presented in Fig. 7.39.a. After the full development of cracks in both directions during the first two cycles the beam showed a near-perfect elastic behaviour. The hysterisis loops are rather small from the 3rd to 7th load cycle. The full plastic rotation was unfortunately not measured at the end of the 7th cycle. The maximum load was maintained for some 30 minutes and during this time additional creep deformations occurred.

Large plastic rotations and the consequent loss of stiffness could be observed during the subsequent cycles. The load-rotation relationship for the 9th to 11th cycles are shown in Fig. 7.39.b. The maximum rotation observed just prior to failure was approximately 18 times the corresponding rotation estimated by the theory based on uncracked section and $4\frac{1}{2}$ times the rotation of the cracked beam.

7.3.6.2 The variation of stiffness.

The levels of the stiffness attained after cracking and after large plastic deformations are distinctly recognisable in Fig. 7.40 which shows characteristics very similar to those of Beam 312.

7.3.6.3 The elongation of the beam.

The curves presented in Fig. 7.41 once more verify the essentially elastic behaviour of the flexural reinforcement during the 3rd to 7th load cycles. The very large yield deformations imposed at the end of the 7th cycle necessitated the deformation history to be presented in two parts. The small permanent elongation after the failure of the beam in the 11th cycle indicates that the failure was due to the destruction of the damaged concrete rather than due to further yielding of the flexural reinforcement.

7.3.6.4 Transverse expansion.

The familiar curves, shown in Fig. 7.42 describe the progress of deterioration. The dotted lines in the bottom diagram indicate that failure was imminent near the right hand support of the beam in the 11th cycle of loading.

7.4 Beam 394

With the provision of heavy web reinforcement it was intended to prevent a diagonal tension failure. Tests of the previous series, described in Chapter 6, showed that the strength of such beams is terminated by failure of the compression zone. An attempt was made to strengthen these areas of the beam, which were subject to large diagonal compression, by placing transverse reinforcement near the four corners. It was also hoped that this confining reinforcement would increase the ductility of the beam.

An overall view of the reinforcing cage for the test beam and its end-blocks is shown in Fig. 7.43. The confining reinforcement was provided in forms of # 2 ties which were continuously bent in a wavy pattern around the stirrups. Extra # 3 vertical bars were provided to improve the anchorage of these ties. A close up of this arrangement is shown in Fig. 7.44. The form of the # 2 ties can be seen in Fig. 7.45 which shows the reinforcement viewed from the top of the beam. The transverse ties correspond to .098 sq.in. steel placed at 2 in. centres horizontally and vertically.

7.4.1 Loading and Testing Procedure

The load sequence for the last beam of this test series was similar to that used for Beam 393. The first four cycles imposed loads within the elastic range. Large loads caused considerable plastic distortions in the 5th and 6th cycles, which were followed by two high intensity but essentially elastic load cycles. At the end of the 9th cycle it was intended to increase the load till failure but the capacity of the testing machine, at 212 Kips, was inadequate. The hydraulic jack was connected to a larger oil pump and after one more intermediate positive cycle the beam was destroyed.

The load sequence is recorded in Table 7.IV.

7.4.2 The Behaviour of the Flexural Reinforcement

The strain distribution along the flexural reinforcement during the first load cycle and the tension force distribution for the top reinforcement show the same features that were observed in previous beams of this series. The respective curves for Beam 394 are shown in Fig. 7.46 and Fig. 7.47. The effect of particularly chosen cyclic loading upon the flexural steel is demonstrated by the strain history of two critical points along the top reinforcement in Fig. 7.48.

This diagram is comparable with Fig. 7.29 prepared from Beam 393. The maximum steel strain, recorded in the bottom reinforcement at the critical section shortly before failure, was approximately 22000 microstrains when the load reached 91% of the theoretical ultimate intensity.

7.4.3 The Performance of the Horizontal Web Reinforcement

The strain distribution for one pair of the horizontal intermediate bars is reproduced only in Fig. 7.49. The typical curves, drawn for a number of load cycles confirm the previous findings, that steel strains are approximately uniform over the whole length of these bars.

7.4.4 The Behaviour of the Stirrups (# 5)

7.4.4.1 The load-strain relationship.

The performance of the stirrups during the 9 cycles of loading is shown in Fig. 7.50. The curves indicate elastic behaviour throughout the instrumented length for the central stirrups. In each of these (No. 6, 7 and 8) the point of maximum strain was chosen. There was no sign of serious deterioration. For this reason the load-strain relationship at several points on these three central stirrups is reproduced in Fig. 7.52 for the whole range of loading, disregarding the fact that they occurred in different cycles. The mean strain for each stirrup agrees well with the ACI relationship.

The behaviour of the outer stirrups was different. This was found also in previous tests. The load-strain relationship at the most highly stressed locality of the two end-stirrups is presented in Fig. 7.51. At this point the positive loading was critical. During negative loads the gauge points were situated in the tension zone of the beam. The signs of distress, visually not observed during the test, are evident at the upper right hand corner of the beam (Gauge location 91) during the 9th cycle. This was the only area where yielding of a stirrup was observed.

7.4.4.2 The strain distribution along stirrups.

Because of the larger diameter bars used, as expected, the strains were more uniform along the stirrups. This is evident from Fig. 7.53 where strain curves for all instrumented stirrups are reproduced for five cycles of the loading. The different behaviour of the outer stirrups is also apparent.

7.4.4.3 The ultimate strength of the stirrups.

The contribution of the stirrups with respect to the shear failure mechanism, associated with separation of the beam into two halves along the main diagonal, is summarised in Fig. 7.54. It is seen that a very good agreement exists with the ACI recommendation in the first 5 cycles of the load. However, towards the end (9th cycle) of the test practically the whole load was carried by the web reinforcement.

The diagonal cracking load was approximately 73^K (v_{dc} = 330 p.s.i.).

7.4.5 Diagonal Concrete Compression

Prior to the application of the intended last (9th) load cycle, preparations were made for a few concrete strain measurements at the critical corners of the beam. Spots were selected on the surface of the concrete which were surrounded by cracks previously formed in both directions. The gauge locations are shown together with the resulting load-strain curves in Fig. 7.55. The gauges were lined up with the estimated direction of the principal compression. Because the crack formation on both sides of the beam was not identical it was only possible to have corresponding gauges attached approximately in the same position. Strains obtained on the East and West face of the beam are reproduced separately.

Because of the inadequacy of the available oil pressure a new cycle (the 10th) was applied in the positive direction. The concrete strains could thus again be followed. The curves in Fig. 7.55 show that at some gauges considerable differences existed between the readings on either side of the beam. Some of the last readings indicated compression strains of the order of 4000 microstrains on the surface. It is to be noted that the plastic compression strains, accummulated during the previous 8 load cycles, are not incorporated in these measurements. It is evident that after the spalling of the cover the diagonal compression strains within the confined concrete must have been very considerable.

7.4.6 The Failure Mechanism

As expected, a shear failure, by diagonal separation, did not occur. The beam failed near the right hand support by diagonal crushing and through sliding shear. The area of the failure is shown in Fig. 7.56. Shortly before failure the cover spalled off and thus the crushed and partly pulverized concrete was exposed, as can be seen in the photograph, Fig. 7.57. Very considerable shear displacements occurred at this stage. It caused the flexural reinforcement, which passed through the compression zone of the beam, to yield because of dowelling action. Some more of the cover was removed after the test so as to expose the main #7 bars in this area. Fig. 7.58 shows a close up of these bars and the large dowel displacements, which resulted from the sliding movements across the compression zone.

7.4.7 Deformations

7.4.7.1 Rotations.

The load rotation relationships are shown in Fig. 7.59 in three groups. The first 4 elastic cycles are positioned in the middle of the figure. The following 4 cycles of high intensity loadings were responsible for some plastic deformations as shown by the right hand diagram. The last positive load cycles are plotted on the left hand end of Fig. 7.59. They show very large yield rotations. The last measurement, taken immediately before failure indicated a total rotation equal to 22 times the theoretical value based on the conventional plastic behaviour of the test specimen. The ductility factor with respect to the cracked test beam was approximately 7. This could have been increased further if a constant strain loading device would have been used.

The beam demonstrated the beneficial effect of the confining reinforcement with respect to ductility.

7.4.7.2 The variation of stiffness.

No large plastic deformations were imposed upon the beam during the second four load cycles. Consequently the loss of stiffness was relatively small as shown by Fig. 7.60, where the variation of the stiffness with cycles loading is recorded.

7.4.7.3 The elongation of the beam.

The performance of the flexural reinforcement is effectively reflected by the load-elongation curves, shown for all eleven cycles of the loading in Fig. 7.61. These graphs show more clearly when large yielding of the flexural reinforcement occurred. The beneficial effect of the heavy web reinforcement and confining steel is demonstrated by the very large elongation of .43 in. before failure.

7.4.7.4 Transverse expansion.

The fully elastic performance of the stirrups preserved the web throughout the test. The small transverse expansions, presented in Fig. 7.62, verify this.

7.5 A Comparison of Deep Coupling Beams

It is necessary to point out that not only the stirrup content was varied in these otherwise identical beams but that the size of the intermediate horizontal bars was also proportionately increased.

7.5.1 The Behaviour of the Flexural Reinforcement

7.5.1.1 Strain distribution along the beams.

A comparison of Fig. 7.14, Fig. 7.26, and Fig. 7.46 shows that the strain curves became gradually steeper as the web steel was increased. This indicates that larger bond forces were transmitted to the flexural bars with heavier stirrups.

The strain difference between the two steel layers at the compression zone of the beam was also reduced as the web steel content increased. The phenomenon was also observed in the previous test series. (See 6.5.1.1).

During the first cycle of loading the outer layers were invariably more highly stressed than the inner bars. The areas of discontinuity at the compression corners are an exception to this.

7.5.1.2 An analytical study of the tension force distribution.

Using the idealised mechanism shown in Fig. 6.105, an explanation was offered in 6.5.1.2 for the unusual pattern of tension force distribution which was observed in these experiments. The same approach may also be applied to the deep coupling beams, but a small modification is required.

In all beams of this series intermediate horizontal bars were also used in the web. Strain measurements indicated that these bars are approximately uniformly stressed over the entire length of the beam proper. It is thus necessary to allow for the contribution of this horizontal web reinforcement when the equilibrium condition is established.

The position of the resultant force, T' , which represents the intermediate bars, is also indicated in Fig. 6.105. Correspondingly Eq. (6.1) needs to be amended by this force thus

$$M = zT + \frac{z}{2}T' + xV_d + x^2\frac{p_s}{2} \tag{7.1}$$

It was pointed out in 6.5.1.2 that the contribution of the dowel force V_d is not significant. The 4 - # 7 bars could sustain a maximum dowel force of approximately 6 Kips before the onset of yielding. This is about 3% of the ultimate load carried by these beams. Consequently the dowel force can be estimated as

$$V_d = \frac{x\,V_{do}}{l\,P_u}\,P = 0.03\,\xi P$$

in a similar way to that adopted for the medium beams.

Because of antisymmetry the tension in the top and bottom flexural reinforcement is the same at midspan, T_c. It will be assumed that the strain in all intermediate bars is the same as that of the flexural reinforcement at midspan. This assumption overestimates slightly the contribution of the intermediate bars. However, as long as intermediate web steel content is considerably smaller than the flexural steel, the error involved is negligible. Hence the force generated in the intermediate bars may be taken as

$$T' = \frac{A_h}{A_s}\,T_c \tag{7.2}$$

where A_h = the total area of the intermediate steel
A_s = the area of the flexural reinforcement in one face of the beam.

It is now necessary to find the value of T_c.

By putting $x = l/2$, $M = Pl/2$, $p_s = V_s/l$, $V_s/P = \eta$ and by combining Eq. (7.1) and Eq. (7.2) it is found that

$$T_c = \frac{2Pl}{z(2 + \frac{A_h}{A_s})} (1 - 0.015 - \frac{\eta}{4}) \tag{7.3}$$

Finally from Eq. (7.1) and Eq. (7.3) the tension force in the main reinforcement is obtained thus

$$T = T_m \left[1 - \frac{1}{a}(1 - 0.015 - \frac{\eta}{4}) - (0.06 + \eta)\xi^2\right] \tag{7.4}$$

where $a = (2\frac{A_s}{A_h} + 1)$ and T_m is the maximum tension generated in the flexural reinforcement.

Using the internal lever arm, z, derived from conventional analysis, (which is known from the experiments to be too large), the tension force distribition was compared for a load level in three beams, when approximately 70% of the shear was resisted by stirrups. The comparison with the measured values is made in Fig. 7.63. The discrepancy in the tension zone of the beam results from the overestimation of the internal lever arm. The contribution of dowel action is seen to be negligible.

7.5.2 The Behaviour of Stirrups

A comparison of beams containing different amounts of shear reinforcement indicated similarity with the medium coupling beams.

The mean stresses for the most critical central stirrups agree well with the AC I proposition. This is shown in Fig. 7.64. However, the maximum stresses are generally higher than the AC I prediction, as indicated by Fig. 7.65. They are influenced by the diagonal cracking load. This is particularly apparent when two identical beams (391 and 392) are compared in which the cracking load differed by 60%.

The excessive stressing of the most critical stirrup need not influence greatly the overall behaviour of the beam.

When the contribution of all instrumented stirrups across the main diagonal was considered, a surprisingly good agreement with the A.C.I. relationship was obtained. This was demonstrated by Fig. 7.8, Fig. 7.36, and Fig. 7.54, which showed that a fraction of the shear was always carried by the concrete mechanisms and dowel action of the flexural reinforcement. Little deterioration was noticeable even after a number of high intensity load cycles as long as the beams were not loaded beyond the elastic limit. Once the flexural reinforcement yielded the stirrups received a considerably larger share of the shear.

Regardless of their size, the stirrups appear to have received the load in all beams at the same rate. The load-stirrup force relationships during the first load cycle assembled for all four beams in Fig. 7.66, confirm this.

7.5.3 Deformations

7.5.3.1 Rotations.

A comparison of the load-rotation relationships in different cycles, as shown in Fig. 7.67, indicate that the increased strength (modulus of elasticity of Beam 392) influenced the stiffness more than did the increased web steel content, (in Beam 393). The marked increase of stiffness in Beam 394 results from additive influence of increased web reinforcement, higher strength and confining reinforcement at the corners.

A more detailed examination of the possible effects of various parameters upon stiffness is presented in Chapter 9.

A comparison of the effect of yielding at the end of a cycle upon the rotations during a subsequent cycle is made in Fig. 7.68.

7.5.3.2 Beam elongations.

There is little difference in the load-elongation characteristics of these beams. The behaviour of the flexural and intermediate horizontal reinforcement, as characterised by the elongation of a beam, is identical in Beams 311 and 312, the slight displacement of the curves, assembled in Fig. 7.69, results from the very different cracking loads in these, otherwise identical beams. The influence of larger intermediate steel content is noticeable in Beam 313 and Beam 314.

The considerations which lead to the establishment of Eq. (7.4), defining the tension force distribution in the flexural reinforcement, may be extended to allow an estimation of the beam's elongation to be made. This was also done for the medium beams in 6.5.3.2.

It may be shown that by integrating the steel strains along the beam proper, the beam's elongation is

$$\Delta H' = \frac{T_m l}{A_s E_s}\left[1 - \frac{1}{a}\left(1 - 0.015 - \frac{\eta}{4}\right) - \left(\frac{0.06 + \eta}{3}\right)\right] \qquad (7.5)$$

Values computed by this equation (7.5) are compared with experimental values in Fig. 7.70. For this comparison typical curves were chosen, which were obtained for conditions that existed after a number of applied load cycles. The figure indicates the relative proportions of slip, elongation of the beam proper and the elastic extension of the reinforcement in the anchorage zones.

7.5.3.3 Transverse expansions.

The stiffening effect of the increased stirrup content is demonstrated by the load-transverse expansion curves, referring to the midspan of the beams, and assembled in Fig. 7.71. The inadequacy of the web reinforcement in Beams 391 and 392 can be seen from the early onset of plastic deformations. This is particularly evident in Beam 392 after cyclic loading.

CHAPTER EIGHT

SHALLOW COUPLING BEAMS

These tests, consisting of four beams designated 241 to 244, were made at the beginning of this research project. Several faults in the instrumentation and testing procedure were identified during this preliminary testing. They were subsequently eliminated so that the testing of the important medium and deep beams could be carried out more successfully with the previously gained experience.

The properties of the beams were summarised in Chapter 5. The span to depth ratio was

$$\boxed{\frac{l}{D} = 2.0}$$

The web reinforcement and the load sequence was varied as follows:

BEAM 241 contained the minimum amount of web reinforcement consisting of # 2 plain bars, with relatively low yield strength. (See Table 5.III). This preliminary test was considered to expose weaknesses of the loading arrangement and instrumentation rather than the behaviour of the specimen. The increments used during the one way loading are recorded in Table 8.I. Using a lower strength concrete a diagonal tension failure was expected.

BEAM 242 was reinforced with # 3 stirrups. It was hoped to attain the theoretical ultimate flexural strength of the specimen in one-way loading. Details of the loading are assembled in Table 8.II.

BEAM 243 was intended to have the same properties as Beam 242. Six cycles of alternating near-ultimate loading were applied during the test as shown in Table 8.III.

BEAM 244 was excessively reinforced with # 4 stirrups and it was subjected to eight cycles of alternating near-ultimate loading as set out in Table 8.IV.

Instead of reporting in detail on the behaviour of each beam, only the significant features of the experimental results are grouped in the subsequent paragraphs for all four specimens.

8.1 Loading and Testing Procedure

The first two specimens were subject to one-way loading, the other two to cyclic loading. Because of the greater flexibility of the beams and particularly because of the large compressibility of the rubber pads (see also 5.4) between the test specimen and the loading frame, larger eccentricities resulted at high load intensities. The point of zero moment moved away from the vertical centre line of the beams, as a consequence of which, considerably larger yield rotations occurred at one end of the beams than at the other.

During positive loading the bending moments at the right hand supports, when the specimens are viewed from the East, could have been 5% higher than at the left hand end. The failure occurred at the right hand support, where larger yield strains of the flexural reinforcement were always observed.

8.2 Crack Patterns and Failure Mechanisms

In Beam 241 diagonal cracks developed from the flexural cracks and fanned out from the compression corners. Up till 95% of the failure load a large diamond shaped crack-free area existed at the centre of the beam, suggesting the presence of a massive diagonal strut. It was found that the

2 plain stirrups, passing through this crack-free area, were stressed to 20-30 Ksi. Failure occurred when this area was split by one large crack at approximately 35° to the horizontal, starting from the upper left hand corner. The failure crack is easily recognised in Fig. 8.1. It is to be noted that this major diagonal crack did not extend from corner to corner of the beam (at an angle of 26.5°) as was the case in the deeper coupling beams.

The crack pattern of Beam 242 was similar. The critical diagonal cracks extending from the compression corners were parallel to each other and had an inclination of 35°, as can be seen in Fig. 8.2. The web reinforcement was sufficient to prevent a separation of the beam into two parts along one of the two major diagonal cracks. Fig. 8.3 shows how the beam failed in diagonal compression at the right hand support.

Beam 243, which was identical to the previous one, showed a similar pattern of cracks at the end of the first load cycle. (See Fig. 8.4). The cracks during the second cycle of loading formed approximately at right angles to the previous ones and were more widely spaced. This can be seen in Fig. 8.5. After 6 cycles of loading the beam failed by diagonal crushing and sliding shear at the right hand support. The yielding of the deteriorated stirrups permitted a shear displacement across the compression zone of the beam. Fig. 8.6, which shows the beam after failure, indicates that few additional cracks developed during the last four load cycles.

Considerably heavier web reinforcement of Beam 244 did not change the crack pattern. (See Fig. 8.7). The number of cracks increased at the right hand support as the 8th load cycle was approached. In spite of large content of web reinforcement, the stirrups in the vicinity of the right hand support deteriorated. The consequent shear displacement and diagonal cracking of the concrete is illustrated by Fig. 8.8.

8.3 The Behaviour of the Flexural Reinforcement

8.3.1 The Distribution of Strains Along the Flexural Reinforcement

Two # 7 bars in one outer layer were instrumented only over the full length of the beams proper. Additional strain readings were obtained at the supports and at midspan. This arrangement did not permit the evaluation of tension force distribution to be determined with sufficient degree of accuracy. However, the pattern of strain distribution along the top or bottom pair of bars indicated a strong similarity with the previous tests.

Fig. 8.9 shows the behaviour of the top two bars in Beam 241. At 26% of the theoretical ultimate load a good agreement existed with the flexural theory. Above this load, with the development of diagonal cracks, the familiar departure from the straight line distribution becomes evident. Failure of the beam occurred before yielding of the flexural reinforcement started.

In Beam 242 yielding extended over a considerable distance from the support when the failure load was approached. This is evident from Fig. 8.10.

Relatively little increase of the strains along the outer bars was observed during cyclic loading in Beams 243 and 244. This is demonstrated in Fig. 8.11 and Fig. 8.12. Only the elastic strains are shown in these diagrams. Already at the end of the first load cycle yielding set in at the critical sections. The subsequent strains in this area, considerably affected by the plastic deformations, are not recorded by curves.

Considerable strain differences existed between the two layers of flexural reinforcement, situated in the compression zones of these shallow beams also. The compression strains at the support section were small in the second (inner) layers.

and at higher load intensities tensile strains prevailed. This is illustrated for the inner top bars at the right hand support of Beam 242 (gauge points 18 and 20) in Fig. 8.10. From this it may be concluded that the "compression reinforcement" contributes little towards carrying compression forces, at least in the first cycle of loading.

8.3.2 Theoretical Considerations of the Tension Force Distribution

The distribution of the tension force along the flexural reinforcement can be predicted by an extension of the study presented for the medium coupling beams in 6.5.1.2. Only the idealised crack pattern, shown in Fig. 6.105.a, need be modified so as to conform with the observed inclination of the major diagonal cracks of the shallow beams. This inclination was found in all usable beams to be about 35° to the horizontal. Therefore Eq. (6.1), which was based upon the equilibrium of the actions shown in Fig. 6.106.b, can be considered as being valid over a distance, ξl, measured from the support of the shallow beams, which is equal to:

$$\xi l = D \operatorname{ctg} 35^{\circ} \approx 34''$$

By allowing for the maximum dowel contribution of the four # 7 bars (approx. 6.0 K) in terms of the ultimate load (approx. 85.0 K) the tension force is found to be

$$T = T_m \left[1 - (0.14 + \eta)\, \xi^2\right] \qquad (8.1)$$

Insufficient instrumentation of the stirrups did not permit the stresses along the major diagonal crack to be determined. Consequently the share, η, of the seven stirrups, which crossed the major crack, could not be evaluated with a satisfactory degree of accuracy.

At yield these # 3 and # 4 stirrups would have been capable of carrying 97% and 139% of the external load respectively. When only 80% of the load is carried across the major diagonal crack by the seven stirrups, i.e. η = .80, Eq. (8.1) indicates that the flexural reinforcement is in tension over 73% to 80% of the span. The latter figure would apply when the contribution of the dowels is ignored. Fig. 8.10, Fig. 8.11, and Fig. 8.12 suggest a good agreement with this prediction after the full development of the diagonal cracks.

8.3.3 Load-Stress Relationship

The stress induced in the flexural reinforcement at the critical support sections are compared with the predictions of the conventional flexural theory in Fig. 8.13. The curves represent the mean stresses determined from strain measurements made on four bars in each of the diagonally opposite tension corners of the beams. It is seen that at higher load, after the development of diagonal cracking, the steel stresses rise more rapidly.

The stresses induced in the 2nd cycle of loading, in Beams 243 and 244, were found to be higher than those in the first load cycle.

8.3.4 Load-Strain History During Cyclic Loading

Fig. 8.14 illustrates the behaviour of the outer layers of flexural reinforcement during cyclic loading. The yield strains imposed at the end of the different cycles is evident. At gauge point 1 the # 7 bars in Beam 244 entered the strain hardening range. It is to be noted that the bars in the compression zone are subjected to small compression strains only during the first load cycle. However, during subsequent load reversal these bars appear to have carried considerable compression forces across the previously cracked compression zone. The compression strains were larger than those observed in the medium and deep coupling beams. This indicates that the flexural behaviour approached that of normal shallow beams.

8.4 The Behaviour of Stirrups

For the first three beams of this series two 4 inch gauge lengths were provided at the centre of every second stirrup. On Beam 244 three gauges were arranged in a similar manner. This proved to be insufficient, for the critical diagonal cracks often crossed the stirrups beyond their instrumented length.

The gauge locations for the first three beams are shown in Fig. 8.15 and for Beam 244 in Fig. 8.18.a. A quantitative comparison of these beams with respect to shear strength is also given in Table 10.I.

8.4.1 The Load-Stirrup Stress Relationship

The stirrup stresses in the lightly web reinforced Beam 241 are shown in Fig. 8.15. The crack pattern associated with these stresses is sketched in the key diagram at the top of the figure. It is evident that the outer stirrups, along gauges 11 and 22, yielded first. In spite of considerable tensile stresses in the central stirrups no visible crack crossed these. At the last load increment, where no strain measurements could be made, a major diagonal crack from the upper right hand corner crossed also the central stirrups, which yielded immediately and thus brought about the failure of the beam. It is to be noted that the development of the shear failure mechanisms in this beam is different from those observed in the deeper beams. In the latter the yielding of the stirrups progressed from the centre of the specimens towards the supports. The AC I relationship, as shown dotted, assessed conservatively the shear strength of the beam. In interpreting the ACI Code, the ratio M/Vd was taken as unity.

Beam 242 and 243 contained a balanced web reinforcement, i.e. the computed ultimate shear strength and flexural strength were approximately equal. Fig. 8.16 indicates that the web reinforcement was sufficient to prevent separation of the beam into two halves along one of the major diagonal cracks. The central portion of the outer stirrups was not highly stressed. In Beam 242 good agreement was attained with the A C I requirements.

8.4.2 The Behaviour of Stirrups During Cyclic Loading

Fig. 8.17.a and Fig. 8.17.b illustrate the behaviour of the companion beam (243) during cyclic loading. These curves demonstrate very clearly that, although the web reinforcement behaved satisfactorily during the first cycle in a similar manner to that observed in Beam 242, a drastic deterioration occurred during subsequent cycles. Some stirrups began to yield already at the end of the second cycle. The contribution of the concrete shear resisting mechanisms gradually diminished with each new load cycle and nearly all stirrups yielded by the end of the 6th load cycle. The large yields near the right hand end of the beam indicated the locality of the failure area.

Beam 244, which was over-reinforced against shear, behaved, as a result of this, more satisfactorily. Fig. 8.18.a and Fig. 8.18.b show that all stirrups behaved elastically, at least during the first four load cycles. The rate of deterioration was much smaller than in Beam 243. The cracked compression zone at the right hand support was, however, not capable of transmitting the large diagonal compression forces without sliding movement. As a result of this the whole shear was thrown on to a few stirrups, which crossed steep, originally minor, diagonal cracks. The gradual overloading of these stirrups, which led to the collapse of the beam, can be observed in Fig. 8.18.b. The phenomenon is very similar to that observed in the deeper beams, which were over-reinforced.

8.5 Concrete Strains

In the preliminary test, the horizontal concrete strains were also measured at mid and quarter span by means of 4 in.

Demec gauges. Two inch gauge lengths were used in the compression zone of the right hand support. The results for this beam (241) are presented in Fig. 8.19. When 20% to 26% of the theoretical load was applied only small flexural cracks were observed near the tension corners of the beam. The remainder of the beam was crack free.

The curves and shaded areas in Fig. 8.19, indicate a marked deviation from the straight line strain distribution at the boundary of the beam. However at 12 inches away from the boundary, at quarter span, the strains agree well with those predicted by the elastic beam theory when applied to the uncracked beam.

With increased load numerous gauges became useless, as they were being crossed by diagonal cracks. The pattern of concrete strains in the diagonally cracked beam again confirmed the previous findings, that over the entire span of the beam the Bernoulli Navier hypothesis breaks down. Considerable compression strains were observed at midspan. This was to be expected from the observed distribution of strains along the flexural steel. At 46% and 72% of the theoretical ultimate load, the compression strains appear to be near linear at the right hand support. The linear extension of these strains would be misleading for it would indicate very considerable plastic strains (approx. 3500 microstrains) in the bottom reinforcement. In fact the beam failed while the flexural reinforcement remained elastic.

In Beam 242 numerous gauges were provided to assess the horizontal and diagonal concrete compression strains - these could be measured up to 97% of the theoretical ultimate load. The localities, at which the strains were measured, are shown in a key diagram of Fig. 8.20. The horizontal compression strains at the right hand support of the beam are also presented in this figure. It is to be noted that some of these measurements were disturbed by diagonal cracks, which deeply penetrated into the compression zone at high load intensities.

The diagonal compression strains, which were measured on Beam 242, at an angle of approximately 27°, are assembled in Fig. 8.21. The nonlinearity of the strains and the predominance of inclined compression across the diagonally opposite corners of the beam was confirmed.

8.6 Deformations

8.6.1 Load-Rotation Relationship

The load-rotation characteristics of the two identical beams (242 and 243) are presented in Fig. 8.22. Significant details of these curves have previously been discussed. It is to be recalled that the rotations at the supports of the beam proper are based on the assumption that the end-blocks are undeformable. In fact rotation occurred also in the end-blocks, so that the actual beam rotations were overestimated by these curves. The latter were evaluated from the measured rotations of the vertical centre lines of the end-blocks.

In the uncracked state the overestimation of rotations is 73% according to the conventional linear elastic analysis. The dotted straight lines represent the theoretical load-rotation relationship, derived from the properties of the uncracked beam proper and adjusted accordingly, so as to allow for the end-block deformations. Thus in the shallow uncracked beams too, a satisfactory agreement appears to exist between the conventional theory and the experiments. In Beam 243 insufficient measurements at low loads enabled only an estimation to be made of the load-rotation relationship.

Fig. 8.23 shows the behaviour of Beam 244 during eight cycles of loading. Owing to different permanent deformations at the ends of the beam, at yield load, the load rotation curves show progressive deviation from symmetrical behaviour, as the number of load cycles was increased.

8.6.2 The Variation of Stiffnesses

Because of lesser number of measurements, the change of the slope along the load-rotation curves could not be followed with the same degree of accuracy that was attained with the deeper beams. For this reason no attempt is made to present the loss of stiffness in graph form, such as was used in Fig. 6.54 or Fig. 7.22. However, to enable an estimation of the reduction of stiffness with cracking and cyclic loading to be made, the values of the maximum stiffness and the elastic recovery in each cycle were collected for the shallow beams in Table 8.V. These correspond with the horizontal regions of the full lines and the dotted lines respectively, in Fig. 6.54 and Fig. 7.22.

8.6.3 Beam Elongations

The load-beam elongation characteristics of the shallow beams, as shown in Fig. 8.24 were found to be similar to those established for the deeper beams. The progressive yielding and accummulation of plastic deformations with cyclic loading is evident. It appears that the frame on which the dial gauges for Beam 243 were mounted may have been accidentally moved by about .015 in. immediately after the zero readings were made. The displacement of the curves (shown by dotted lines) during the first two load cycles suggests this. The characteristics of the curves for Beam 243 did not seem to have been affected by this at all.

CHAPTER NINE

AN ANALYTICAL ASSESSMENT OF THE DEFORMATIONS OF COUPLING BEAMS

9.1 The Distribution of Strains Along the Flexural Reinforcement of Coupling Beams.

With the presentation of the experimental results, an attempt was also made to explain the reasons for the very marked difference between the strain patterns or tension force distributions obtained from the elastic beam theory and from these experiments. In 6.5.1.2 and 7.5.1.2 equations were proposed which predict, with a satisfactory degree of accuracy, actual forces in the flexural reinforcement after cracking. These equations take the geometry of the beam, the contribution of the vertical and horizontal web reinforcement into account. They can be suitably interpreted when the placing of the reinforcement is being detailed or when further computations are required to assess other aspects of structural behaviour.

9.2 The Elongation of the Beams

The quantitative determination of the tension force distribution along the coupling beam lead also to the assessment of the elongation of the beam proper. The appropriate expressions were given in 6.5.3.2 and 7.5.3.2.

The experiments showed that the relative displacement of the end-blocks is considerably larger than the elongation of the coupling beam because of the elastic and permanent deformations within the anchorage zones of the flexural reinforcement.

In Chapter 3 the elastic behaviour of two coupled shear walls was presented and the effect of cracking was examined. It is probable that the elongation of the coupling beams is significant enough to affect this behaviour, at least in the lower third of the structure. However it should have no effect upon the ultimate strength that the structure could develop.

9.3 The Estimation of the Stiffness and Coupling Beams After Cracking

The observed distribution of concrete and steel strains, the variation of stirrup forces and the crack pattern indicated that the load-deformation characteristics of coupling beams can not be assessed with the conventional techniques of the elastic beam theory. It is evident that the cracked beam forms a mechanism in which the flexural reinforcement, the stirrups and the concrete blocks formed between cracks interact with each other in a complex manner. So far the characteristics and the relative importance of the numerous components have not beenexplored satisfactorily.

The significance of the dowel action of the flexural reinforcement has been studied more closely only recently.[68] The attention was focussed upon the behaviour of the "concrete struts" which are formed between diagonal cracks of a beam by Fenwick[60], who showed that, apart from axial compression, these struts or "teeth" are also subject to aggregate interlock forces, shear forces and bending moments.

In attempting to evaluate at least in an approximate manner, the deformation characteristics of coupling beams, it is necessary to neglect a number of actions. In coupling beams dowel and aggregate interlock actions may be considered as having only secondary importance. Neither of these have yet been fully evaluated quantitatively. Observations made during this project suggest that the more important modes of load transfer and the associated distortions in cracked coupling beams could be identified as follows:

i.) Shear transfer by truss action and consequent web distortions.

ii.) Arch action and compression across the main diagonal of the beam.

iii.) Flexure and associated rotations.

iv.) Tying action of the flexural reinforcement and consequent elongation of the beam.

With certain simplifications the load-deformation characteristics of each of these modes may be approximated and hence by superposition the stiffness of the cracked coupling beams can be estimated.

9.3.1 Shear Deformations Owing to Truss Action

A considerable portion of the shear force is transferred from one support of the beam to the other by the stirrup reinforcement, which, together with the diagonal "concrete struts", forms a truss. The crack pattern of the test beams suggests that this truss is different from the one which has traditionally formed the basis of the conventional stirrup design. Instead of being parallel the compression members radiate from diagonally opposite corners of a beam. The struts formed by an idealised crack pattern were illustrated in Fig. 6.105.

Members of the analogous model truss are shown in Fig. 9.1.a. The web consists of vertical tension members (stirrups) and radiating compression members (concrete). For the purpose of this analytical study the tapered struts are replaced by prismatic members, the depths of which are equal to the depths of the tapered ones at mid-height of the beam. (See Fig. 9.1.b).

It is evident that the shear forces, V_s , may be transferred through as many paths of this statically indeterminate truss (Fig. 9.1.a) as there are stirrups. The deformations resulting from one component, S, of the total shear, V_s, may be evaluated with the aid of Fig. 9.2. This diagram shows the principal dimensions, the vertical forces and displacements of a typical linkage ABCD. The total shear displacement at D, relative to A, is made up of the axial strains of the three web members thus:

$$\Delta_v = \Delta_1 + \Delta_s + \Delta_2$$

It follows from first principles that

$$\Delta_1 = \frac{2Sl_1^4}{sbE_c l_s^3}\,;\quad \Delta_s = \frac{Sl_s}{sbE_c np_w}\,;\quad \Delta_2 = \frac{2Sl_2^4}{sbE_c l_s^3}$$

Hence by adding the component displacements and expressing the position of the stirrup by the distances x_1 and x_2, the shear displacement of the particular linkage becomes:

$$\Delta_v = \frac{2Sl_s}{sbE_c}\left[\left(1 + \frac{x_1^2}{l_s^2}\right)^2 + \left(1 + \frac{x_2^2}{l_s^2}\right)^2 + \frac{1}{2np_w}\right] \tag{9.1}$$

It also follows from the requirements of strain compatibility that the vertical displacements of all linkages at the right hand support must be the same. (i.e. Δ_v must be the same for any associated value of x_1 and x_2). This is only possible if the stirrup force intensity varies along the span. It is easy to show that the stirrup forces must be large at midspan and smaller at the supports. The assumptions and approximations made do not warrant an exact determination of the stirrup force intensity.* For this reason further studies will be based on a parabolic force distribution, the maximum stirrup force being at midspan. Experimental evidence, such as shown in Fig. 6.72, reinforces such an assumption. Consequently stirrup elongations will be large at midspan and

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* An exact analysis in the form of a mathematical exercise is possible.

smaller at the support. Numerous transverse expansion curves, such as Fig. 6.56, Fig. 6.79, Fig. 7.24 and Fig. 7.62 also verify this.

Instead of using discrete stirrup forces it is more convenient, for the purpose of analysis, to express them as a force per unit length of the beam, i.e.

$$p_x = \frac{S}{s}$$

These are distributed along the beam in a parabolic form, as shown in Fig. 9.3. The maximum, p_c, minimum, p_o, and average, p_s, values of p_x have a known relation to each other, so that

$$p_o = \frac{3p_s}{1 + 2\frac{p_c}{p_o}} \qquad (9.2)$$

To satisfy the compatibility requirement one may chose any two linkages which, according to Eq. (9.1), should give the same shear displacement, Δ_v. This equation may be expressed in a more general form thus:

$$\Delta_v = \frac{2l_s}{bE_c} p_x f(x_1, x_2) \qquad (9.1.a)$$

The two linkages of the analogous truss can be conveniently chosen so that, according to Fig. 9.2, the position of the vertical member in one is defined by

$$x_1 = 0 \quad \text{and} \quad x_2 = 1 \quad \text{when} \quad p_x = p_o$$

and in the other linkage by

$$x_1 = x_2 = \tfrac{1}{2} \quad \text{when} \quad p_x = p_c$$

Consequently the compatibility requirement may be stated as follows:

$$\Delta_v = \frac{2l_s}{bE_c} p_o \, f(0, \tfrac{1}{2}) = \frac{2l_s}{bE_c} p_c \, f(\tfrac{1}{2}, \tfrac{1}{2})$$

Hence

$$\frac{p_c}{p_o} = \frac{f(0,1)}{f(\frac{1}{2},\frac{1}{2})} = \frac{1 + (1+\nu^2)^2 + \frac{1}{2np_w}}{2(1+\frac{\nu^2}{4})^2 + \frac{1}{2np_w}} \tag{9.3}$$

where $\nu = \frac{l}{l_s}$

This enables the degree of non-uniformity of stirrup forces, as expressed by the ratio p_c/p_o, to be determined. It may be shown that the ratio of the maximum stirrup stress to the minimum one varies between the following approximate limits:

$$1 < \frac{p_c}{p_o} < 1.75 \quad \text{when} \quad \frac{l}{D} = 1 \quad \text{and} \quad 0 < np_w < \infty$$

$$1 < \frac{p_c}{p_o} < 2.70 \quad \text{when} \quad \frac{l}{D} = 1.5 \quad \text{and} \quad 0 < np_w < \infty$$

In fact the ratio is likely to be larger because the wall elements, adjoining the coupling beams, impose restrictions upon the boundaries and do not permit the last stirrups at $x_1 = 0$, $x_2 = 0$ to be appreciably strained. Curves, showing the transverse expansions of the beams indicate this trend.

By combining Eq. (9.1.a), Eq. (9.2) and Eq. (9.3) and by putting

$$x_1 = 1, \; x_2 = 0 \quad \text{when} \quad p_x = p_o, \; \text{and} \quad p_s = \frac{nP}{l}$$

the shear rotation of the analogous truss is obtained as follows:

$$\theta_v = \frac{\Delta_v}{l} = \frac{6\eta P \left[1 + (1+\nu^2)^2 + \frac{1}{2np_w}\right]}{\nu b l E_c \left(1 + 2\,\frac{p_c}{p_o}\right)} \tag{9.4}$$

9.3.2 Shear Deformation Owing to Arch Action

According to the assumptions, previously stated, the external shear force not resisted by truss action, V_a, must be transferred across the beam by means of diagonal compression. The diagonal strut, its equivalent estimated dimensions and its deformation are illustrated in Fig. 9.4.

Accordingly, the mean area of the tapered diagonal strut is

$$\frac{lb}{4l'}(3D - 2z)$$

and the diagonal compression force is

$$\frac{l'}{z} V_a = \frac{l'}{z}(1 - \eta)P$$

The shear rotation caused by diagonal shortening is therefore

$$\theta_a = \frac{\Delta_a}{l} = \frac{4(1 + \nu_1^2)^2(1 - \eta)P}{\nu_1^2 b(3D - 2z)E_c} \tag{9.5}$$

where $\nu_1 = \frac{l}{z}$.

9.3.3 Flexural Rotations

The Bernoulli-Navier hypothesis not being applicable, a new definition of the rotation of a particular section need be introduced. A plane section across the beam may be replaced by a similar plane which passes through two points located at the top and bottom reinforcement.

Fig. 9.5.a shows a typical undistorted coupling beam and a number of equidistant vertical planes at right angles to the plane of this beam. In the diagram below, Fig. 9.5.b, a typical distribution of the tension force in the top reinforcement, T_t, and the bottom reinforcement, T_b, is also reproduced. As a result of the accummulated strains along the flexural steel the originally vertical planes become inclined

with respect to the centre section, which may be taken as a reference. The inclination or rotation of these planes is shown qualitatively in Fig. 9.5.c. It is thus possible to define and evaluate the rotation of any section of a coupling beam. The maximum rotation at the supports of the beam is, with reference to this latter diagram,

$$\theta_m = \frac{\Delta_s' - \Delta_s''}{d - d'}$$

where

$$\Delta_s' = \frac{1}{A_s E_s} \int_o^{\frac{l}{2}} T_t \, dx$$

$$\Delta_s'' = \frac{1}{A_s E_s} \int_{\frac{l}{2}}^{l} T_b \, dx = \frac{1}{A_s E_s} \int_{\frac{l}{2}}^{l} T_t \, dx$$

so that

$$\theta_m = \frac{1}{A_s E_s (d - d')} \left[2 \int_o^{.5} T(\xi) \, d\xi - \int_o^{1} T(\xi) \, d\xi \right]$$

where $T(\xi)$ was defined by Eq. (6.3) and Eq. (7.4).

When the dowel forces are neglected and no intermediate horizontal steel is used the flexural rotation becomes, after evaluation of the integrals:

$$\theta_m = \frac{\eta P l^2}{8 A_s E_s (d - d') z} \qquad (9.6.a)$$

As expected flexural rotation increases proportionally with the percentage of the load, η, which is being resisted by stirrups. The larger is the share of the web reinforcement the larger bond forces can be generated (at the node points of the analogous truss) and thus the differential elongation of the flexural reinforcement increases.

When intermediate bars are also used the rotation is arrived at by modifying the above equation so that

$$\theta_m = \frac{\eta\, T_m\, l}{4A_s E_s (d-d')} \qquad (9.6.b)$$

where T_m is the maximum tension force induced at the supports.

9.3.4 Rotation Owing to Beam Elongation

The elongation of the flexural reinforcement over the entire length of the beam proper is associated with diagonal compressions resulting from truss and arch actions. Both of these require a tying action of the flexural reinforcement. The rotations caused by the diagonal concrete compression strains have already been evaluated.

In Fig. 9.5.b the dotted line indicates the lengthened top reinforcement. The length of the compression diagonal is shown to remain constant. Hence the resulting rotation is

$$\theta_l = \frac{\Delta_l}{l} = \frac{\Delta_s'}{(d-d')}$$

where $$\Delta_s' = \Delta_s' + \Delta_s'' = \frac{1}{A_s E_s} \int_0^l T(\xi)\, d\xi \qquad (6.7)$$

From Eq. (6.3) and Eq.(7.4), which defined $T(\xi)$, and Eq. 6.7 the rotation owing to the elongation is obtained thus

$$\theta_l = \frac{T_m l}{A_s E_s (d-d')} \left[1 - \frac{1}{a}\left(1 - \frac{\eta}{4}\right) - \frac{\eta}{3}\right] \qquad (9.7)$$

In this expression "a" becomes infinity when no intermediate horizontal bars are used in the beam. (Ref. 7.5.3.2).

9.3.5 Total Rotation and the Proportions of the Component Rotations

To obtain the stiffness of a coupling beam it is necessary to evaluate its total rotation. By the superposition of the previous four cases

$$\theta_B = \theta_v + \theta_a + \theta_m + \theta_l \qquad (9.8)$$

An examination of these expressions revealed that the amount of web reinforcement does not substantially affect the total rotation and hence the stiffness of a coupling beam. However, the stirrup content drastically affects the mode of shear resistance and the relative proportions of the consequent component rotations.

To illustrate this observation the component rotations are presented for a typical medium (312) and a deep (393) beam. It was assumed that the contribution of the web reinforcement, η , in carrying the load across the main diagonal crack, varied between zero and 100%. Fig. 9.6 shows that with decreased contribution of the web reinforcement the mode of shear resistance changes from truss into arch action. The flexural rotations are insignificant but the elongation of the beam, particularly for beams with a larger span to depth ratio, is a major source of beam rotations.

A coupling beam in which less than one half of the load is resisted by stirrups is likely to be impractical and it should certainly not be used for earthquake resistant construction. This range is shown by dotted lines in Fig. 9.6.

The relative proportions of the component rotations, in terms of the total rotation of a beam, are presented in Table 9.I for three in each of the two major groups of test beams. Only the practical limits of η (.7 to 1.0) are considered in this table. The combined influence of the stirrup content and concrete is expressed by the parameter np_w . It may be noted that the shear rotation decreases with increased stirrup content, (i.e. more effective truss action) and that the flexural rotations increase because of the larger bond forces developed in the beam.

9.3.6 A Comparison with Experimental Results

In Fig. 9.7 a comparison of the equations of component rotations and the experimentally obtained load-rotation relationships is presented for the medium and deep coupling beams.

The diagram on the left hand side of this figure (a) shows the absolute stiffness of the beams. This is defined as the load, P , required to cause unit rotation of a beam at both of its supports. The load corresponds with the bending moment pattern of a typical coupling beam or lamina. To enable a comparison to be made it was necessary to apply a correction to the experimental rotation values, presented in the previous chapters. It was pointed out in 6.1.5.1 that the beam rotations evaluated from dial gauge readings were based on the rigid body rotations of indeformable end-blocks. As these end-blocks are in fact distorted the computed experimental rotations over-estimate the rotations of the beam proper. As a corollary to these, the experimental curves previously presented underestimate the stiffness of the beam proper. The discrepancy is large when both, the end-block and the beam, are uncracked. With cyclic loading, extensive cracking and yield deformations the stiffness of the beams reduce rapidly, while the stiffness of the end-blocks is little affected. When applying these corrections it was assumed that the end-blocks remain crack free. Consequently the end-block deformations have relatively little effect (5-12%) upon the stiffness of the cracked beam. Numerous load-rotation curves indicate that the so adjusted theoretical relationship, based on the conventional beam theory, agrees satisfactorily with the measurements when the test specimens were free of cracks.

The heavy lines at the top of the diagram (Fig. 9.7.a) show the stiffness of the coupling beams in the uncracked state. The importance of shear deformations is apparent. The stiffnesses of the deep beams are approximately 35% larger than those of the medium beams. The increased depth however should account for 115% increase of the flexural stiffnesses.

The lower part of the diagram (Fig. 9.7.a) shows the absolute stiffness of the cracked beams. For each, the theoretical and the experimental stiffnesses, a range of values is indicated. The band of theoretical values results from the assumption that the participation of the stirrups, η , may be taken to be between 70% and 100%. The range for the experimentally determined stiffnesses, on the other hand, represents the highest and lowest values observed during cyclic loading. These observed values correspond with the "horizontal tops" of the load-stiffness curves, such as shown in Fig. 7.22. The dotted lines indicate the maximum stiffnesses observed during unloading. This occurred normally at the end of the first or second cycle of loading.

Considering the approximations, which formed the basis of the previously presented analytical study, it may be said that fair agreement exists between theory and observation. The theory overestimates the stiffness of the medium beams and slightly underestimates that of the deep beam. The percentual difference between the stiffnesses of the cracked medium and deep beams is about the same as in the uncracked state.

The diagram shows, what is important, that all beams suffered a dramatic loss in stiffness. The loss is more apparent in Fig. 9.7.b, where the stiffnesses of each cracked beam is presented as a percentage of its stiffness in the uncracked state. It may be said that irrespective of the amount of web reinforcement and the beams geometry, the stiffnesses of these coupling beams were reduced at least by 80%. After high intensity alternating loading the maximum stiffness of the beams is approximately 15% of the original value. It is to be noted that the maximum stiffness in any one load cycle was considered in these diagrams. This gives no measure of the overall performance of the beam after cycles loading, for it ignores the "soft range" through which the previously overloaded beam must pass at the beginning of a new cycle. (See for example Fig. 7.21).

CHAPTER TEN

CONCLUSIONS AND RECOMMENDATIONS

10.1 The Elastic Behaviour of Coupled Shear Walls

The complete solution of the "laminar" shear wall analysis, based on the Beck-Rosman approach, was presented for two common forms of structures. These were subjected to a seismic type of lateral static loading. The assumptions of this elastic analysis were critically examined and attention was drawn to the significance of cracking in the assessment of the appropriate stiffnesses.

In the example of an 18 storey building, it was shown that owing to the cracking of the coupling system the critical wall moments can increase by 50% or more.

10.2 The Elasto-Plastic Analysis of Coupled Shear Walls

On the assumption that the coupling beams are the first to attain their ultimate capacity, a step by step approximate procedure was developed, by which the actions and deformations of a coupled shear wall structure can be traced till a collapse mechanism is formed. A bilinear elasto-plastic behaviour of all members formed the basis of the analysis.

An advantage of the analysis is that it yields the requirements for laminar ductility. In an illustrative example it was shown that for a particular 18 storey shear core the critical coupling beams need to possess a ductility factor of 27 if the overall plastic behaviour is to correspond with a ductility of 4.

The limitations of the analysis lie in the assumptions made with respect to the ability of the principal structural members, in particular the coupling beams, to undergo the necessary plastic deformations without loss of resistance.

10.3 Experimental Evidence on the Behaviour of Coupling Beams.

10.3.1 Flexure

10.3.1.1 The distribution of the stresses along the flexural reinforcement of diagonally cracked coupling beams drastically deviated from the imposed bending moment pattern. In medium and deep coupling beams tensile stresses were generated over the entire span in both, the top and the bottom reinforcement. As a consequence of this, none of the steel in these doubly reinforced concrete coupling beams can be relied upon to carry compression forces. Similarly the beneficial effect of the "compression reinforcement" upon the ductility of the beam must be discounted.

10.3.1.2 After the full development of cracking, the critical steel stresses were higher than those predicted by the conventional flexural theory. Because of the interaction of shear and flexure, the internal lever arm reduced progressively as the ultimate load was being approached. As opposed to the behaviour of normal reinforced concrete beams, in coupling beams the internal compression resultant was found to move away from the compression edge of the beam at the attainment of the ultimate load.

10.3.1.3 The reduction of the internal lever arm and the consequent reduction of the ultimate strength was accentuated after cyclic loading. Tensile yielding imposed upon the reinforcement in one cycle was not recovered after a load reversal of equally high intensity. Plastic steel deformations in tension, caused by alternating loading, were largely accumulative in both faces of the beams.

10.3.1.4 The discrepancy between the (conventional) theoretical ultimate flexural load and the failure load increased with decreasing aspect ratio (l/D) of the beams. This is apparent from a quantitative comparison of the test beams, as presented in Table 10.I. In shallow coupling beams the ultimate flexural capacity was satisfactorily predicted by the customary (Whitney) theory.

10.3.2 Shear

10.3.2.1 Measurements verified the phenomenon, also observed by others, that stresses in a stirrup vary considerably along its length. Local high stresses, however, do not necessarily affect the ultimate shear strength. The latter depends on the combined strength of all stirrups which cross a potential failure crack.

10.3.2.2 The stirrup reinforcement did not behave according to the laws of the conventional truss analogy. In medium and deep elastic coupling beams, a statically indeterminate truss carried the major portion of the constant shearing force. In such a truss the compression struts radiate from diagonally opposite corners of the beams proper. Each diagonal strut has a different inclination and thus, within the elastic limit, each symmetrically situated stirrup carries a different share of the load.

10.3.2.3 In beams, underreinforced for shear, failure occurred by separation into two equal halves along the main diagonal. The yielding of the stirrups progressed from the centre of the coupling beams towards their supports.

10.3.2.4 The strength of the web reinforcement was determined by the number of stirrups encountered by the potential failure crack along the main diagonal. For beams with an aspect ratio (l/D) of 1.4 to 2.0 the critical diagonal crack was formed at an inclination of approximately 35°. For medium and deep coupling beams the diagonal tension (separation) failure occurred along the diagonal connecting opposite corners.

10.3.2.5 The shear force sustained by the coupling beams, without the assistance of the web reinforcement, was satisfactorily predicted by the present A C I Code. (See Table 10.I). It is to be noted that the beams of this investigation would be classified as deep beams, for the analysis of which most codes make special and only vague recommendations.

10.3.2.6 When shear governed the strength of a coupling beam the A C I Code satisfactorily and conservatively estimated the combined shear strength of the concrete mechanisms and the web reinforcement for monotonous static loading. (See columns 6, 9 and 11 of Table 10.I.)

10.3.2.7 The contribution of the shear resisting mechanisms in coupling beams, other than the web reinforcement (i.e. arch action, dowel and aggregate interlock forces), rapidly diminish after a few reversals of high intensity cyclic loading, particularly when yielding of the flexural reinforcement has also occurred. The strength of stirrups in beams, which have not been web reinforced for the full ultimate shear, deteriorate rapidly during cyclic loading.

10.3.3 Concrete Stresses and Strains

10.3.3.1 As expected, the nonlinear distribution of the concrete stresses at the encastre boundaries of the uncracked coupling beams was clearly evident.

10.3.3.2 The complete break down of the Bernoulli-Navier hypothesis in the diagonally cracked coupling beams was verified.

10.3.3.3 The pattern of concrete stresses conformed best with that which could be derived from the behaviour of the analogous truss. The critical compression stresses were always inclined.

10.3.3.4 The flexural reinforcement in the compression zones of the beams interfered with rather than assisted in the carrying of the internal compression forces.

10.3.3.5 Once subject to tensile yield, the flexural reinforcement retarded the closure of cracks in the compression zone after load reversal and thus made the surrounding concrete less effective in transmitting compression stresses. A reduced internal lever arm of the resisting forces was one of the consequences of this phenomenon.

10.3.4 Horizontal (Intermediate) Web Reinforcement

10.3.4.1 Because the flexural reinforcement was subject to tension over the entire span, excepting shallow coupling beams, the loaded beams have become longer. Therefore all intermediate horizontal bars became engaged in load transfer. The stresses over the length of these bars were approximately uniform. They increased the ultimate moment capacity of the medium and deep coupling beams.

10.3.4.2 The horizontal web reinforcement tended to produce a finer mesh and a more uniform spacing of cracks.

10.3.4.3 The extensive yielding of the principal flexural reinforcement and the consequent plastic deformations of the coupling beams were arrested by the intermediate bars, when the latter operated within the elastic limit. Also they appeared to have slightly increased the stiffness of the cracked beams.

10.3.5 Confining Reinforcement

Confining reinforcement, consisting of closely spaced transverse ties in the corners of a deep coupling beam, together with heavy stirrup reinforcement, prevented the rapid deterioration of the beam after cyclic loading and considerably increased its ductility.

10.3.6 <u>Deformations</u>

10.3.6.1 The rotations of uncracked coupling beams could be approximated with a satisfactory degree of accuracy using only the classical flexural theory, i.e. neglecting the deep beam effects.

10.3.6.2 The deformations of cracked coupling beams were significantly influenced by the biaxial state of strains, i.e. horizontal and vertical steel tensile strains and diagonal concrete compression strains. In contrast to slender flexural members the elongation and transverse expansion of these beams were considerable. Consequently the deformed shape could no longer be defined in terms of the shape of the beams' axes.

10.3.6.3 Transverse expansion measurements indicated best the imminence of distress, particularly after cyclic loading.

10.3.6.4 Extensive rotational measurements revealed three distinct ranges of a beam's behaviour after it was subjected to a number of high intensity load reversals.

a.) A "soft range" at very low loads. This was associated with the closure of the previously formed cracks. Its extent was proportional to the total previously imposed plastic deformations.

b.) A "steady range" at medium loads with the characteristics of linear elastic behaviour.

c.) A "plastic range" at near ultimate load resulting from the yielding of the flexural and/or shear reinforcement.

The change from one range into another was gradual.

10.3.6.5 In terms of support rotations relatively small ductility could be observed in all beams. However, it is very probable that considerably larger plastic rotations occurred than the measured ones. In supplying inadequate information on ductility this project, through the use of unsuitable loading arrangement, failed to fulfil one of its principal aims.

10.3.6.6 Deflection observations indicated that shear distortions greatly overshadowed flexural ones.

10.3.7 Stiffness

10.3.7.1 The loss of stiffness, after the cracking of coupling beams, was considerably more than the loss encountered with normally proportioned reinforced concrete flexural members. The greatest reduction of stiffness occurred after the formation of diagonal cracks.

10.3.7.2 As long as beams behaved within the elastic range, a number of high intensity load reversals did not significantly alter their stiffnesses.

10.3.7.3 A few excursions into the postelastic range of behaviour resulted in a further loss of stiffness. At this stage the stiffness measured the beam's resistance after it moved through considerable displacements ("soft range") at low loads.

10.4 Analytical Studies of Coupling Beams

10.4.1 Tension Force Distribution and Beam Elongation

10.4.1.1 By satisfying the equilibrium requirements in an analogous beam, which was modelled on the cracked real beam, the distribution of the tension force generated along the flexural reinforcement could be determined. The equations, in which the contribution of the stirrup reinforcement (η) was the major parameter, agreed well with the experimental evidence.

10.4.1.2 The equations of steel force intensity allowed the elastic elongation of the beam proper to be computed.

A comparison with test measurements revealed that slip and elastic deformations along the anchorages of the flexural reinforcement in the end-blocks, were each of the same order as the elongation of the coupling beam proper.

10.4.2 <u>The Determination of Stiffness</u>

10.4.2.1 Using the model of an analogous truss the component rotations owing to truss action, arch action, tying action of the flexural reinforcement, and flexure were analytically approximated. From these the stiffness of the cracked coupling beams was approximated. No attempt was made to introduce empirical parameters. A fair agreement was found to exist between theory and experiments.

10.4.2.2 The study also indicated that although the component rotations may be considerably affected by the share of the stirrups in resisting shear (η), the total rotations do not depend upon stirrup performance.

10.4.2.3 The analysis showed that in medium and deep coupling beams the flexural rotations accounted for 10% to 15% of the total rotations, the remainder being entirely due to shear deformations.

10.5 <u>Suggestions for Future Research</u>

10.5.1 To supplement the experimental findings of this project it would be useful to repeat some tests using a displacement controlled load device, so as to enable the full plastic range of the coupling beams to be evaluated.

10.5.2 It is possible that a concentration of stirrup reinforcement adjacent to the built-in supports might be useful to prevent or to delay the destructive shear displacements across the cracked compression zones of coupling beams, which are subjected to high intensity cyclic loading. Suitable tests should reveal whether this can be achieved with only the varied spacing of the web reinforcement or with the use of additional stirrups at the supports.

10.5.3 The effectivness of a diagonal arrangement of the principal reinforcement and its influence on post-elastic performance of coupling beams should be investigated within the limits of constructional possibilities.

10.5.4 The plastic behaviour of walls under the action of bending moment, shear and axial tension, and a suitable arrangement of the reinforcement to improve upon ductility need be fully explored.

10.5.5 The applicability of an elasto-plastic analysis, based upon a realistic approximation of the true behaviour of all structural members, would need to be verified on small scale reinforced concrete coupled shear wall model structures.

10.6 Design Recommenations

10.6.1 The Design of Coupling Beams

10.6.1.1 The ultimate flexural strength of medium* and deep* coupling beams can be assessed with the use of the established principles of the ultimate load analysis in reinforced concrete, with the following restrictions:

a.) Only 90% of the so computed flexural resistance is to be taken into account.

b) All reinforcement in the compression zone of the beam is to be ignored in the strength computations.

c.) When high intensity cyclic loading is to be expected, as in earthquakes, a further 5% reduction of flexural capacity is to be allowed for.

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* According to the terminology of this project.

10.6.1.2 To ensure the utilisation of the maximum possible ductility, the shear strength should not be permitted to govern the design. At least a "balanced design" must be aimed at in which the shear strength provided matches, but preferably exceeds, the ultimate strength in flexure. In earthquake resistant design therefore, the stirrup reinforcement, crossing the potential failure crack, should carry at least the full shear corresponding to the ultimate moment capacity. The potential failure crack forms across the main diagonal of the beam or at an angle of not less than 35° to the horizontal. Preferably additional stirrups should be provided in the immediate vicinity of the encastre supports of the beam to give restraint to the compression zone of the beam during alternating loading.

10.6.1.3 Horizontal intermediate web reinforcement should be used in deep beams for crack control. Any such reinforcement, situated in the tension zone of the critical section, should be included in the evaluation of the ultimate flexural capacity.

10.6.1.4 When an elasto-plastic analysis so indicates and when it is feasible from the point of view of construction, confining reinforcement, such as ties extending between the legs of stirrups, may be used in the four compression corners of the critically situated coupling beams.

10.6.1.5 All horizontal reinforcement should pass through the full span of a coupling beam without cut-offs or laps, and be generously anchored into the adjoining walls. It is to be noted that the concrete in the anchorage zones may be subject to transverse tensile strains and may thus contain cracks parallel to the reinforcement to be anchored.

10.6.1.6 Flanges from slabs, that may frame into coupling beams, should be ignored in the strength computations. All flexural reinforcement should be accommodated within the stirrups of the beams so as to enable the most effective development of a truss action.

10.6.1.7 Because of the unique distribution of the tension forces, the nominal flexural bond stresses are not likely to be critical in the design of coupling beams. However, if the bond performance needs improving it should be achieved with the use of smaller diameter bars and not through the provision of additional flexural reinforcement.

10.6.1.8 The displacements imposed upon earthquakes are often large enough to force a member, such as a coupling beam, into the postelastic range. To be able to assess with reasonable confidence the magnitude of actions so generated, it is important to be aware of the true properties of the materials at the time of the design. It may be a fallacy to believe that a low yield strength adopted in the design calculations necessarily leads to a safe design.* In a so designed coupling beam, large earthquake induced displacements may generate bending moments, at the true yield strength of the flexural reinforcement, which induce shear forces, large enough to destroy the beam in a brittle manner.

10.6.2 Coupled Shear Wall Structures

10.6.2.1 The elastic behaviour of two coupled shear walls may be conveniently assessed by the "laminar analysis". For the results, such as displacements, to be realistic the loss of stiffness, caused by cracking of the structural members, must be considered. In a normal structure the coupling beams are likely to be the first ones to crack.

* The current standard building by-law in New Zealand specifies a yield strength of 33000 p.s.i. for mild steel. The reinforcement so marketed, and also used in this investigation, has a yield strength of approximately 45000 p.s.i.

With the use of the equations, presented in Chapter 9, the stiffness of the cracked coupling beams can be satisfactorily evaluated. The approximate reduction of stiffness was 80% in terms of the stiffness of the uncracked medium and deep coupling beams. This dramatic loss suggests that the present practice of shear wall analysis is likely to considerably overestimate the stiffness of coupled shear wall structures. This may lead to adverse dynamic design properties.

10.6.2.2 Even with its limitations, the proposed elasto-plastic analysis (Chapter 4) indicates

a.) the individual requirements of structural ductility of various members when a specified overall ductility is to be attained in earthquake resistant design,

b.) the likely magnitude of the maximum load upon the coupled shear wall structure when all its reserves are exhausted.

10.6.2.3 The knowledge of the ultimate load on the shear wall structure enables a comparison to be made with the capacity of its foundations. For cantilever structures, such as coupled shear walls, stability requirements may be the ones that determine the limits of lateral load carrying capacity. The accummulation of the theoretical ultimate actions at the base of the coupled walls may produce moments at foundation level, well in excess of the minimum moment which would cause the structure to overturn. When the lateral load on the structure is limited by stability requirements at the foundations - a frequent situation in the design of multistorey structures - a major part of the coupled shear wall may operate in the elastic range. In such cases the demand for large ductility may be considerably reduced.